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THE MECHANICS OF INTRUSION OF THE BLACK HILLS (S. D.) PRE-CAMBRIAN GRANITE

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The Pre-Cambrian rocks of the Black Hills consist of a great series of slates and schists, for the most part monotonously alike, striking in a northwest direction and having steep dips generally, except in the extreme southwest, to the east. Close study shows that the persistent eastward dips represent both schistosity and bedding, the two for the most part parallel; and that the series is compressed into a number of great folds which comprise innumerable minor isoclinal folds. A sufficient number of individual beds have been traced to locate the position and nature of the greater axes of folding, and to locate two important faults.

To the south great intrusions of granite break through the strata, and around the principal mass, forming Harney Peak, a notable schistosity is developed parallel with the granite contact, and superimposed upon the normal schistosity of the region.

Field relations indicate that this granite came into its present position in the main by physical distension of the invaded rock body, under great load, and that it thus modified to an important degree the normal process of regional compressive folding, forcing the schists into closely appressed recumbent folds, parallel with the advancing surface of the magma. The rocks were deformed; they yielded to the advance of the magma, and the schistosity produced by this movement and folding lent itself to the further injection of the granite by numberless parallel dikes and by lit-par-lit intrusion. The harder rocks, the quartzites, were distended and broken apart by the upward movement, and the magma flowed in between the blocks.

The relation of dikes to schist layers, the fact that lit-par-lit injection tends to neutralize chemical and physical differences between the magma and the invaded rock, the probable low temperature of the granite, and the fact that positive evidence of absorption or assimilation on a large scale is lacking, all support the belief that physical displacement and not assimilation was the primary process by which the forward movement of the magma was accomplished. The possibility that large blocks of roof in the areas of harder quartzite have been engulfed in the magma, and that this process was in these areas an important one, must remain an open question. In the main it is considered that the magma actually displaced the rocks either along great rifts or by pushing the yielding softer rocks before it into recumbent folds.

ON THE FOSSIL ALGAE OF THE PETROLEUM-YIELDING SHALES OF THE GREEN RIVER FORMATION OF COLORADO AND UTAH

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In the region extending from northwestern Colorado west into Utah and north into Wyoming, there are great areas of, generally, carbonaceous shales, which, in places, exceed 3000 feet in thickness. Scientific, as well as economic interest has been aroused in these shales, because they have recently been discovered to yield petroleum when subjected to destructive distillation in closed retorts. These shales are typically dark brown in color, very fine grained, hard, firm, tough and compact, except where weathered into thin, more or less curled and brittle laminae. Surfaces long exposed to the action of the weather bleach to whitish or yellowish gray. Some beds in the series of these shales already examined are so highly carbonaceous that they closely resemble compact lignite in appearance and burn readily when heated; other beds contain rather high percentages of finely divided mineral matter.

When freshly broken, the rock gives off a distinctly bituminous odor, but, so far as observed, it contains no free oily compounds, although nodules and particles of a substance resembling asphaltum, but insoluble in the usual solvents of that substance, are sometimes found; and, as already noted, petroleum is always found among the products of its distillation.

Careful study of the stratigraphic relationship of these shales by the U. S. Geological Survey, has shown them to belong to the Green River

Formation of Eocene time. This formation is widely known in some of its Wyoming phases for its richness in fossil fish remains.

About a year ago, samples of the shale from near De Beque, Colorado, and, later, from other localities, were submitted to me for microscopic examination. Inspection led to the conclusion that the rock was composed so largely of organic residues, that it would be practicable to apply to it methods of sectioning already in use for making thin sections for the microscopic study of peats and coals.

After some experimental work, it was found possible so to soften the material without any visible change in its original form or structure, that after it was imbedded in paraffin, sections could be made from it with razor or microtome as readily as from properly prepared fresh vegetable tissue. Sections of any desired thinness can thus be made, and in any quantity. Moreover, these sections can be so treated that when mounted they can be studied with the highest powers of the compound microscope.

Several highly interesting and important facts have been discovered by examining specimens of the shale prepared as outlined above. The ground mass of the sections was found to be of the nature of a somewhat granular, organic jelly, closely resembling in optical properties, some of the structureless, colloidal or sapropelic peats, solidified and compacted into a dense, tough, impermeable magma.

Apparently the hardening and compacting of this material from its original gelatinous condition to its present state, was a process of slow dehydration and contraction, without any violent disturbance or action of any outside metamorphosing agencies. This is shown by the undisturbed position and structure of the shale beds in the mass, as well as by the almost entire lack of visible changes in the delicate plant structures that are often abundant in the sections examined.

The ground mass, magma, or body of the shales, from its structure as seen under the microscope, seems to have been originally vegetable matter. It apparently was a mixture of finely divided and flocculent plant debris and living microscopic plants, bacteria, etc., most of which probably were more or less saprophytic, decomposing and still farther disintegrating the mass of detritus in and on which they lived.

Imbedded in this magma, as many of the sections showed, are innumerable plant cells, structures, and entire plants, almost as perfectly preserved as if they had been prepared by the most refined methods of the plant morphologist. Many of these fossils are spores, fungi or structures belonging to the higher plants, but a large percentage of them are clearly Algae of low types. Three distinct types of Algae have been discovered by the work so far done.

1. A very considerable number of cellular, filamentous and gelatinous forms, which, from their methods of reproduction and general morphology, are clearly to be placed with the Blue-Green Algae—(*Myxophyceae* or *Schizophyceae*). These include forms that show close relationship to modern types of these low Algae, not only in size and vegetative structure, but also, many of them show a series of reproductive stages so characteristic as to fully establish their taxonomic relationships. In one instance at least, a minute but highly characteristic blue-green Alga has been found that can almost certainly be referred to a living genus, namely *Spirulina*. This fossil specimen is so well preserved and the measurements and form of the plant are so closely in harmony with those given in descriptions of one of the species of the genus, that one is tempted to identify the fossil as belonging to the same species, notwithstanding the hundreds of thousands of years which must have elapsed since the Eocene time. Other fossil species that may be referable to existing genera are not uncommon.

2. The Yellow-Green Algae (*Chlorophyceae*). Somewhat less common than the low forms already mentioned are plants that can be referred with more or less certainty to the Green Algae. Among these, both the cellular and filamentous types have been discovered, and, in a few cases, the identification may be said to be positive. Among cellular forms, is one in particular that so closely resembles a species of *Pediastrum* in all essentials, that its reference to this or a closely related genus is clearly not an error.

Among the filamentous fossil forms classed as Green Algae, a type has been noted with spiral chloroplasts, so like those which characterize the well-known *Spirogyra*, that it appears at least to be a prototype, if not strictly a species of this genus. Other filamentous fossil species have been found which, because of structural peculiarities, can be safely placed among the Green Algae.

3. Unclassified Algae. A large number of plants have been found, which, from their form and structure, appear to be Algae, but which as yet, cannot be classified among known living Algae. Some of these unclassified types are the largest plants so far found in the shales, and others are very minute. Some of the more noteworthy of the large forms seem to have been thin, gelatinous films growing on the surface of the detrital material which is now the imbedding magma.

A very common and puzzling plant is one of the gelatinous film-like forms of considerable size, which seems to be without cellular structure, but which, nevertheless, has small areas on its surface which on magnification disclose definite, usually oval, spots covered with a net-like

pattern formed by small crowded hexagonal depressions on the surface of the film. The meaning of these netted areas has not yet been worked out, but it seems possible that the structure is a reproductive surface from which spores arise. The whole plant after most careful examination seems to have been made up of gelatinous material forming a thallus, with no definite divisions into cells.

Besides the filmy gelatinous types, there are large numbers of seemingly gelatinous forms that were more or less filamentous. These types vary greatly in the length and width of their filaments, which are often more or less flattened, and which branch in an eccentric way, quite unusual among the modern Algae. The filaments lack visible divisions into definite cells and seem to have been wholly gelatinous or to have been sheathed by thick gelatinous envelopes, which have obliterated the evidences of cell division.

It is well known, however, that we have very slight and indefinite knowledge of the floras of present-day fresh and salt waters, especially of those which grow in and around the deeper parts of waters in which there are deposits of organic origin, and many of the plants which have been described from shallow waters are so imperfectly known that their life histories have never been worked out. These facts make the study and classification of fossil forms much more difficult than it would otherwise be, and their relationships harder to determine.

Careful study of the material under discussion indicates how perfectly plant remains, originally of the most delicate structure, have been preserved. Even very thin-walled and fragile cells and filaments show by internal evidence that they did not even collapse as they were incorporated into the mass in which they are preserved. Contraction, caused by shrinkage from the loss of water in the original substances, and the later changes which have transformed them into rock, has manifestly occurred. Dehydration has been so slow and regular, however, that it has produced no changes that have in any way interfered with the microscopic study of the properly prepared material. The perfection of the preservation of the plant remains in these rocks is such that it is quite evident that, with such specimens to work with, there need be no difficulty in learning the facts or in fully and correctly interpreting them. All doubt as to the possibility of Algae, and similar cellular plants of most delicate structure being preserved and their remains becoming a conspicuous part of the organic matter found in carbonaceous rocks is removed by the studies here recorded.

The discovery of this interesting flora of minute fossil Algae, doubtless the most extensive of its class yet reported, embracing as it does thou-

sands of individuals and an undetermined number of species, many of which have not yet even been photographed, is in itself a noteworthy matter. The discovery seemingly has a more important bearing, however, on the broad geologic problem of the origin and development of petroleum and related carbonaceous compounds, than it has on that of the development of plant life.

Several steps looking towards the solution of this great genetic problem have been taken already and tentative advances have been made. The structure of the rock, the kind of fossils in it, and their state of preservation, make it possible to determine, to some extent at least, the sources of the abundant carbonaceous matter which it contains, and the conditions under which this matter accumulated. The state of decomposition of the original organic substance contained in the magma has, to some extent, to be assumed, since it is so largely structureless. It is practically certain from studies made on peat, that the breaking down of the original structure of this material is rather a physical than a chemical change, a comminution, not a decomposition. The true chemical condition of the organic matter of the rock is more correctly interpreted from the fossils, which probably are approximately at the same stage of carbonization as the structureless magma. The chemical condition of the finely preserved fossils, therefore, becomes highly significant, this, judging from their color, physical condition, etc., is indicative of very early stages of carbonization, which in turn, shows a very limited action by the transforming agencies originating within the earth itself, i.e., those producing metamorphism.

Finally, and of great importance in the consideration of the larger problem, is the determination of the actual substances now existing in the rock itself, from which the distillates obtained by heating, are derived. It will readily be seen that this constitutes a somewhat prolonged investigation but it is easily within the reach of modern chemical science.

It can already be stated positively that the organic matter of these shales, so far as examined, in very large part is of vegetable origin, with but very slight traces of the remains of animal organisms. The plant remains were accumulated in water of considerable depth and purity and in places apparently remote from shore lines. To a considerable extent this vegetable matter seems to have been of the nature of drift material or plankton, although it is largely mixed with the remains of Algae and similar plants that certainly grew upon and in the magmatic mass as a place of attachment, if not actually aiding in its decomposition by their growth.

The beds thus formed were built up very slowly, and when completed and solidified were so compact and fine grained as to be nearly or quite impermeable to liquids from the outside. If infiltrated by any material from without before the water originally included was drained and dried away, it is difficult to see how this could be done by liquids lighter than water, coming into it in the form of a uniform seepage from below. Exactly contemporaneous with the gradually progressive disappearance of the water and the chemical changes accompanying this, the sediments were compacted, solidified and lithified to their present texture and condition, and it seems highly probable that penetration by liquids from any outside source during the induration would be increasingly difficult. If these views are correct, the gaseous and liquid compounds obtained by distillation from the shale are not of external origin, but result from the decomposition of the carbonaceous material laid down during the formation of the rock. The absence of demonstrable liquid hydrocarbons in the shale itself would tend to strengthen this view, although not an insurmountable objection.

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ARCHEOLOGICAL EXPLORATIONS AT PECOS, NEW MEXICO

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During the past summer the Department of Archaeology of Phillips Andover Academy began the excavation of the ruined Indian Pueblo of Pecos in San Miguel County, New Mexico. It was the wish of the Department to undertake the thorough exploration of a site large enough and of sufficient scientific importance to justify work upon it for several years.

Pecos lies in the well-watered and fertile upper Pecos valley at an elevation of about 7000 feet. The buildings are located on the summit of a flat-topped ridge of sandstone slightly elevated above the surrounding plain, and consist of two house-clusters containing together, as nearly as can be made out in their present state of ruin, about 1100 ground floor rooms. A few hundred feet to the south stand the roofless walls of the large Spanish mission church.

The place was discovered by Coronado in 1540. At that time it was the most populous of the Pueblo villages, being able to muster 500 fighting men. The chroniclers of the expedition describe the town as a quadrangular structure, four stories in height, on the balconies of which

one could make the entire circuit without putting foot to ground. The people were brave, prosperous and skilled in all the arts of the Pueblos. After Coronado, Pecos was visited by several other explorers and when New Mexico was actually settled by the Spanish at the beginning of the seventeenth century Pecos was made the seat of a mission, a large church was erected and the town became one of the most important and prosperous in the province. During the eighteenth century, however, its position on the eastern frontier exposed it to ever-increasing raids by the Comanche. Thus worn down and greatly reduced by sickness, the Pecos people managed to hold out until about 1840 when, numbering only seventeen souls, they abandoned their home and put themselves under the protection of the Jemez, a nearby tribe related to them by language and tradition.

We have, then, a historically recorded occupation from 1540 until 1840; and an examination of the potsherds scattered about the ruins shows also practically all the types of prehistoric wares known to occur in the upper Rio Grande district. The Department therefore believed that the site had been settled in very early times and that people had lived there, presumably continuously, through all the intervening centuries. So long an occupation does not seem to have occurred at any other place in New Mexico available for excavation and it was hoped that remains would there be found so stratified as to make clear the development of the various Pueblo arts and to enable students to place in their proper chronological order numerous New Mexican ruins whose culture has long been known but whose relation one to another has been entirely problematical. This hope was strengthened by the fact that Mr. N. C. Nelson of the American Museum, New York, had recently discovered very important stratified remains at San Christobal a few miles to the west and it was believed that, if similar deposits could be found at Pecos, they would serve not only to complement and, so to speak, cross-reference the finds of Mr. Nelson, but also to carry the story of Pueblo arts from 1680, the date of the abandonment of San Christobal, down through the very little known eighteenth century to the middle of the nineteenth, from which time to the present day it may still be recovered by studies among the living tribes. These were the chief reasons for the choice of Pecos.

The Regents of the State Museum of New Mexico, in whom the title to the ruins is vested, requested in making their very generous offer of the site, that the old church, greatly damaged by the weather and by vandalism, and in imminent danger of entire collapse, be so repaired and protected that no further harm should ensue. This work

was cheerfully undertaken and was carried out during the summer by Mr. J. L. Nussbaum of Santa Fé; the body of the church was cleared, the walls strengthened and underpinned, and fallen portions of several arches, doorways and windows were replaced.

The archaeological work consisted entirely of excavations in the great rubbish heap on the east side of the mesa. Digging was commenced there because it was necessary to clear a place in which earth and stone, removed from the houses above, could later be dumped without danger of covering unexplored ground. This portion of the work continued longer than was expected because of the unforeseen depth and

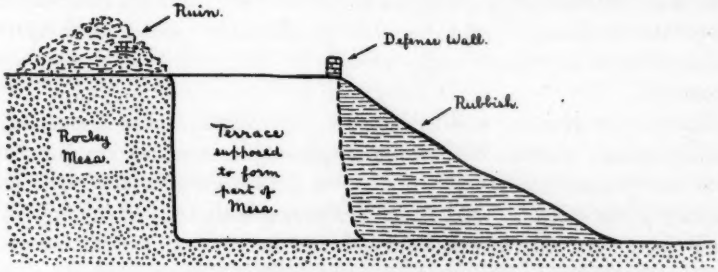

FIG. 1. SUPPOSED EXTENT OF RUBBISH BEFORE EXCAVATION

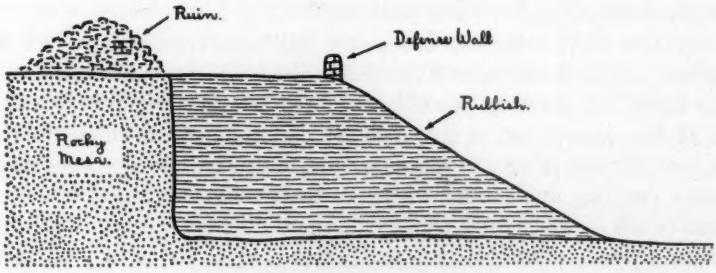

FIG. 2. ACTUAL EXTENT OF RUBBISH

extent of the rubbish deposit. This miscalculation of the mass of the rubbish was due to the mistaken belief that the terrace surrounding the building proper formed part of the solid rock mesa (fig. 1). When, however, the trenches reached the defensive wall supposed to mark the end of the midden, it was found that the terrace, instead of being rock, was composed entirely of rubbish (fig. 2) thus approximately doubling the amount to be handled. Further complications were encountered in the form of buried kivas and old defensive walls, so that at the end of the season the main trench face was still some distance away from the

mesa. Three narrow exploring trenches, however, were pushed through the terrace to the rock wall.

Against the mesa the deposit was found to vary from 15 to 20 feet in depth. It sloped away gradually toward the plain. Its composition was very uniform, soft earth darkened by charcoal and decayed organic matter, filled with potsherds, animal bones and chips of building stone. The heap was also used as a cemetery, our trench revealing over 150 burials; the skeletons lay, for the most part, in the flexed position, at depths varying according to the period of interment of from a few inches to 17 feet. Mortuary pottery occurred only in the earlier graves and not abundantly even in them. Besides skeletons and pottery there were recovered a great number of broken and many whole specimens of bone and stone tools, as well as ornaments, clay toys and figurines.

The most important results, however, were, as had been hoped, of a stratigraphical nature. It became apparent, as soon as digging had gone far enough into the refuse heap to permit observations, that the pottery at the bottom was markedly different from that at the top and that there were several distinct types between. Data on this subject were collected to some extent by examination and note taking along the face of the trench, but principally by tests made at different points as the work advanced. The tests consisted of the collection of all the sherds in a given column of débris, the fragments from each layer being placed in a separate paper bag bearing the numbers of the test and of the layer. In the early tests the face was laid off arbitrarily in 1 foot or 18 inch layers; but as it was found that this method often resulted in the splitting of strata, the later tests were not laid out beforehand and a new bag was started whenever there was encountered a zone of sand or ash or when there appeared a sherd different in type from those above. Care, of course, had to be exercised to choose for the tests places which seemed to contain no burials, and all trials were abandoned when it became clear that a grave shaft had disturbed the original deposition of the refuse.

The mass of stratigraphic material thus gathered could not be worked over in the field; final results, therefore, are not yet available. It may be said, however, that the earliest or lowest type was the black-and-white. Above this occurred a long series of wares with glazed ornamentation, running from an early type with clear cut geometric ornament in sharp glaze, up through more developed stages having heavier glaze and conventional ornament, to a late and degenerate style in which the glaze paint was so poor that the decoration lost all form and became

little more than daubs and splashes. This degenerate stage extended well into historic times, as was proved by the finding, in that horizon, of European objects, but before the final abandonment of the site there occurred a sort of renaissance, when glaze painting was given up and there was produced a yellowish pottery excellently finished and tastefully decorated with dull black pigment.

The details of this long ceramic history; the growth, for example, of vessel forms; the developments in the cooking wares; the influences from without that undoubtedly produced some of the changes; all these must await closer study and more data.

The stratigraphy of the site, then, throws invaluable light on the local culture; it also may be expected to teach us much as to its external relations. In the first year's work there has been found pottery of the historic period from the Hopi villages, Acoma, Zuni, and Jemez, as well as prehistoric wares from the Little Colorado, Lower Gila (?), and San Juan. From the East we have pottery, buffalo-scapula hoes and snubnosed scrapers; from the South, clay bells and spindlewhorls strongly Mexican in type. Such finds as these give an indication of what important results may confidently be hoped for. If we can definitely recognize and chronologically arrange the successive culture stages at Pecos, we can extend that knowledge and thus fit into their proper chronological order the one-culture ruins that abound in the Rio Grande. We may also hope to learn, from trade objects found at Pecos and in the chronologically arranged one-culture ruins, the relative age of many other groups, not only in the Southwest but even well beyond its borders.

MAN AND METALS

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In the course of a long-continued study of the uses of fire by man, I have recently completed a review of the branch referring to metallurgy, the results of which are embodied in the following brief summary.

The free metals which early man might have found and used are copper, gold, and silver. It is known that he used copper, and to some extent silver and gold, and that he worked them as other hard substances, becoming gradually acquainted with the property of ductility of these metals. Since copper was the most common metal in a free state and potentially at an early stage the most useful, it is safe to say that its employment was the beginning of the age of metals. Strictly,

the age of metals begins with the working of these naturally occurring substances by means of heat. The development of metallurgy has been the development of heat, resulting in remarkable inventions and achievements, and in the increase of temperature from that of the open camp-fire to the fierce heat of the blazing arc. Fire itself, the heritage of man through all ages, though adapted mostly to the advancement of man himself, had very limited use in the early periods, but the reactions of man on fire and of fire on man must have been profound.

Metallurgy is impracticable without a concentration of heat beyond that furnished by a fire in the open supplied with limited fuel. The first and natural step was to confine fire to a limited space demarked with stones or within a shallow earth-basin. Observations would show that the heart of the fire possesses greater heat than the margin and in an especially hot focus, one day when the time was ripe, a metal, likely copper, was tried and was found to melt and run without losing its structure. From such an event as that fancied here metallurgy may have had its crudest beginnings. The objection is noted that copper which liquidifies at 1083°C. would not melt ordinarily in an open-hearth fire under natural draught and this will be commented upon later in describing the efficiency of a smothered fire massed in an earth pit. Early experiments however were far from an established practice of metallurgy, a vast time may have elapsed before metals could be reduced from the ores. The history would begin with free metals regarded as stone and continue with metals (copper) hammered into shape; smelting of free metals (copper), and casting, followed by work in combined stone and metal technique; alloys both adventitious as in tin-bearing copper and designed as in the addition of cassiterite or oxide of tin to copper producing bronze.

This imperfect résumé covers a great advance in heat production to the time when a fourth metal, tin, becomes known to man. The bronze age with its ramifications of art, of science, and of material and social progress can only be characterized as a period when the cruder tentative essays in metals were brought to a somewhat exact metallurgical formula of pre-science. The Bronze Age also illustrates the vast importance of alloys. It more especially marks for the purpose of this paper the production and regulation of heat sufficient to smelt copper and tin forming the alloy known as bronze.

It may be possible to trace the steps by which this heat was reached and it seems probable that its attainment depends on some other use of fire rather than for metallurgy. In this regard attention is called to the confinement of fire in a primitive oven for baking pottery or the

cooking of vegetal substances, both very ancient arts and nearer to early needs than the working of metals. The cooking of vegetables in its primitive stage gives rise to ovens which may be ancestors of the furnace for reducing metals, namely the pit or 'gipsy' oven in which refractory fruits and vegetables were cooked. These pits were dug in the earth and heated by burning fuel in them, the coals removed and the vegetables put in and covered up, a fire often being set on top. It is observed that in modern instances the heat produced by a fire of this character is very great at the bottom of the glowing coals. There is a great concentration of heat due to the confinement of the fire in a non-conducting medium and this effect does not depend greatly upon excess oxygenation of the fuel by draught, in fact too much draught cools a fire. In such a fire bronze may easily be melted. There is used to this day a pit-smelter for melting bronze and this device would seem to be a survival. I saw one of these ovens in use in Washington a few years ago where it was employed for melting small portions of bronze for minor castings. European discoveries indicate that bronze was melted in pits in the earth and that the bronze founders had also acquired a knowledge of refractory clays which may have been contributed to some extent by the potter's art. It is evident that clay and metal working are intimately connected.

The production of higher temperatures depends upon the oxygenation of the fuel beyond that ordinarily furnished by natural draught. Natural draught in a forced fire fed with abundant fuel may give a central area of high temperature, but the question of fuel supply would militate against the common employment of this method. The simple way in the earlier periods would be to aerate the fire with a fan-like hand-blower such as are used with the braseros and small cooking fires in many parts of the world. There would follow other devices to produce draught, culminating in the modern tremendous mechanical development for forced draught. Some steps on the way may be observed in the aboriginal pit ovens for cooking previously mentioned. Most of these are without draught holes or any device for producing draught, depending for their utility upon the absorption and retention of heat in the earth wall of the pit. The pit ovens of the Pueblo tribes have a draught flue alongside the pit. A step in advance involved in building separate fire containers is the raising of the fire and the production of bottom draught. To forced draught however we must look for practical advance in metallurgy, and in comparatively recent times the use of preheated draught or blast was epoch making in the science of the production of metals. The forerunner of this was the tuyere embedded beneath the fire.

The knowledge of metals in the Bronze Age was confined to those native metals, copper, gold, and silver, and of reduced metals tin and possibly lead. The development of temperature in the Bronze Age paved the way for the reduction of iron, the most important metal known to man.

In general no extensive reduction of ores could be attained with the facilities of the Bronze Age. It is improbable that much copper ore was reduced in this age, the source of supply being nuggets of native copper. Tin in the form of the oxide resembles a native metal and would attract the attention of man. It is easily reduced at a moderate heat and requires no roasting or flux, but whether this had any bearing on the discovery of bronze is not clear. The combination of copper and tin presents no difficulties since both metals are easily accessible and of wide occurrence. Native copper containing tin and forming a natural bronze is known, but is rare and had no effect on the discovery of alloys. Copper alone is very difficult to smelt and cast so that there is no very definite copper age and not many cast artifacts of this metal have been found. The copper age would be the period when native copper was shaped by hammering as especially in North America. Bronze may have resulted from experiments with mixtures of various substances to lower the melting point and to admit of casting in a closed mould. As a rule ores consisting of oxides of metals presented little difficulty in smelting. Carbonate ores could be oxidized by roasting before smelting following the old process. The chief difficulty in the reduction of ores is due to the presence of impurities chiefly sulphur and phosphorus which remain the bane of modern metallurgists. Another difficulty is the kind and proportion of flux. The question of fuel was not pressing, charcoal being the usual and immemorial supply up to the use of coal.

The niceties of the reduction of ores remained for modern science, early metal workers confining their attention to varieties which could be reduced by the facilities and knowledge in their possession. It is possible that the bronze age even at its focus may have overlapped the iron age and it is not strange that some students should have been led to assert that iron preceded bronze. The accumulated knowledge required for the reduction of an inconspicuous ore to secure a metal not known in a free state and with properties and value unknown is greater than the production of an alloy of two metals one free and the other practically free and both known to man for untold generations. There are also the high temperature, 1200–1300°C., and the experimental data on fluxes required in the reduction of iron ore. The metallurgy of iron was a distinct advance on that of bronze and made use of the ex-

perimental knowledge and mechanical equipment acquired in the bronze age. The bellows which classical writers attribute to Daedalus and Anacharsis may have made possible the reduction of copper ores in the later bronze age and may have unlocked at another period the coming metal iron.

The Iron Age, which began before the dawn of history, ushered in the fifth metal whose effect on the material world was destined to be incalculable. Man had seen metallic iron derived from meteorites and there exist implements made from this material, but its occurrence was sporadic and limited, so that it had no effect on his arts. Heavy and lustrous hematite ores were also widely used for implements and ornaments. When and in what locality iron began to be smelted is unknown, but as has been stated, it is the logical successor to the advanced technique of the metallurgy of bronze. Primitive iron working may still be observed in Africa and has been described in India. African ore is an oxide comparatively very easy of reduction and ore beds are of general occurrence. The smelter consists of a basin-shape depression in the ground beaten down hard and leading to the center is a clay tuyere with which the rude bellows are connected. The ore is heaped with charcoal in this depression and in the midst lighted coals are placed and covered over. The bellows are started and after a time the ore is reduced to a fluxed mass in which small pellets of iron are found. The mass is pounded up and the iron sorted out to be hammered into larger pieces by a second process. This was essentially the method pursued in India. Only limited amounts of metal were secured. Casting of iron was unknown in Africa. The knowledge of smelting is quite diffused in Africa so that there are no chief centers of manufacture and distribution, although some tribes are famed as blacksmiths and a few tribes depend on alien artificers for their iron utensils. The smelting of iron ore is impossible without forced draught. The bellows therefore is a device of great interest and importance. The history of draught-producing devices embraces the hand fan, the tube for blowing, the air bag in which the air is captured and forced out by pressure through a tube, the double air bag to promote a steady stream of air, the double air bag worked by rods, in which a valve appears, the double plunger bellows, the piston bellows, the folding bellows with organ valve used by blacksmiths before the invention of the fan blower, and the fan blower which brings back the primitive hand fan much improved.

The most marked types, the complete knowledge of which would cast much light on the history of Western and Eastern foci of metallurgy, are the air bags and piston blower the former belonging to the Europe-Africa province and the latter to Malaysia.

Furnaces also have an instructive development. There may be cited the rudimentary heap-furnace with base draught and rude bellows; the pit furnace for melting bronze with fan draught; the prehistoric earth wall furnace for reduction of copper and iron ores probably with bellows draught; and the tall furnace built of masonry and called Catalan furnace with means for forced draught.

The furnace of the Iron Age has been well described by archaeologists for northern France, Switzerland, Austria and other localities in Europe. On account of convenience they were located on the vein of ore and in forests which would furnish charcoal. The furnace consisted of a pocket excavated in the side of a clay bank lined with clay and, when charged, covered with clay leaving a flue above and draught hole below. The ore and charcoal were packed in layers and the draught was probably supplied with rude bellows.[1]

There is a considerable history, which may only be indicated, in regard to the tools required in metallurgy, as those for handling—pincers, tongs, shovels; for shaping—hammer, file, rubbers, polishers; for casting—crucibles with supports, handlers and poker-stirrers, moulds of stone, metal, clay and artificial mixtures; for finishing—file, abraders, polishers, punch, drill, chisel, rivets, etc.

At some period in the Iron Age it was discovered that iron could be cemented or case hardened by reheating with organic materials away from air, the process forming a layer of steel over the softer iron or, if continued, converting the whole mass to steel. In some localities, possibly India, at remote times metallurgical processes had given rise to steel. This new alloy could be tempered, a quality foreign to any other metal known to man. Up to comparatively recent times however steel had a limited use, but at present among civilized nations steel is more common than iron.

An interesting survey is presented of the state of metallurgic art in the various world areas. It shows that the Pacific Islands, most of the Americas, Australia, and much of Asia are in the premetallic stage; Malaysia is in the beginning of the metallic age by acculturation, the first demand being weapons; native Africa is advanced in iron metallurgy, using two metals; civilized (Mediterranean) Africa advanced in the use of metals; Europe (Mediterranean) shows early development and use of five metals. There are four great ancient foci of metallurgic art; Southern Europe, Northern Africa, Western Asia, and Eastern Asia. The latter is of doubtful origin and affiliation, but the other foci were connected. American foci are Central America, Mexico, Peru, and Wisconsin.

Ancient man beginning with one important and two minor free metals acquired by heat tin and iron giving him an acquaintance with five metals. Modern man has freed 52 more, knows 85 elemental substances and predicts the eventual discovery of several others.

[1]Rollain, A., Scories de fer antéhistoriques., *Bull. Soc. Anth., Paris*, 4 s, 1899, p. 318. Discussion by M. Lionel Bonnemère.

ON THE OBSERVED ROTATIONS OF A PLANETARY NEBULA

By W. W. Campbell and J. H. Moore

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The geometric forms of certain classes of nebulae are such as to suggest that they are in rotation about axes passing through their effective centers. We refer especially to the spiral nebulae and to those so-called planetary nebulae which are of ring, circular, or elliptical form with relatively condensed or stellar nuclei.

In the latter part of 1915 we tested several planetary nebulae by means of observations made with the Mills 3-prism spectrograph and obtained positive evidences of rotation, as Doppler-Fizeau effects.

The planetary nebula No. 7009 in Dreyer's New General Catalogue, right ascension 20h. 58m., illustrated herewith, was submitted to the test of four spectrograms. In each case the slit of the spectrograph was placed centrally across the image of the nebula and made slightly longer than the diameter of the elliptical outline of the nebula, and the condensed nucleus of the nebula was kept central in the slit during the exposures. In two exposures the slit was placed upon the longer axis of the nebular image, parallel to the slender rectangle drawn above the nebula in the illustration to represent the slit in length and direction. In a third exposure the slit was placed east and west across the image. On these spectrograms the bright lines of nebulium (4959 and 5007A) and of hydrogen (H Beta), comprising the recorded nebular spectrum, were inclined to the 'zero' direction, as indicated by the comparison spectra of hydrogen, helium and titanium on the same plates. This inclination of the lines is illustrated (exaggerated) by the direction of the bright line on the diagram of the slit in the upper part of the figure. The section of the lines corresponding to the western parts of the nebula are displaced to the violet, and the sections corresponding to the eastern parts, to the red. Interpreted as Doppler-Fizeau effects, the western parts of the nebula are approaching us and the eastern parts are receding from us by virtue of the rotation of the nebula. The fainter ends

of the lines do not maintain the inclination of the bright central section of the lines but turn backward slightly. We interpret this to mean that the corresponding faint outer strata of nebulosity do not rotate with speeds proportional to the radii of the strata—relatively to the speeds of the bright inner strata—but that they 'lag behind.'

The maximum displacements of the nebular lines correspond to points in the nebula 9 or 10 seconds of arc west and east of the central nucleus, and the observed component of rotational speed is of the order of 6 km. per second.

On a fourth spectrogram, with the slit upon the minor axis of the elliptical image, the nebular lines are not inclined to the 'zero' direction, and they therefore fail to indicate any components of rotational motion in the plane passing through the minor axis of the nebula and the observer —a result in accord with expectations.

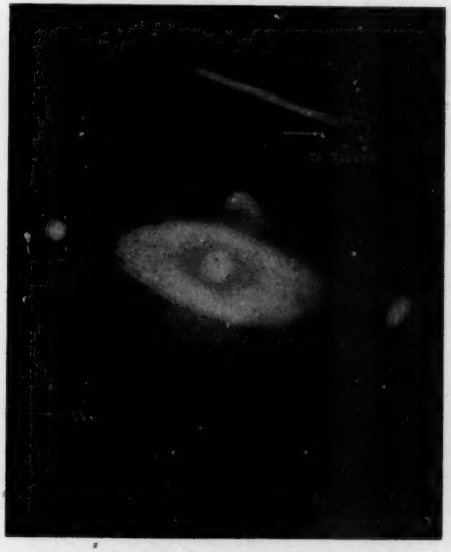
The observations which we have described, considered in connection with the geometrical form of the nebular image as photographed, leave essentially no room for doubt that the nebula is rotating about an axis through the central nucleus approximately at right angles to the plane passing through the observer and the major axis of the image, the nebular materials lying nearest to us being carried from west to east by the rotation.

Measures of the rotational velocity of the nebula enable us to draw some interesting conclusions concerning its mass. If we assume that the axis of rotation is located as described above, then the orbital speed of the nebular materials lying at a distance of 9 seconds of arc from the center is about 6 km. per second. If we provisionally assume the mass of the central nucleus to equal that of the Sun, Kepler's law connecting the periodic time with the distance from the nucleus tells us definitely that the nebula is distant from us only 8.9 light years. This must be regarded as an improbably small value, in view of other evidence bearing on the question. For assumed distances of 100 and 1000 light years, which we have reason to believe are more probable orders of nebular distance, the masses of the nebula would be respectively 11.3 and 113 times that of the Sun, and the corresponding periods of rotation 1371 and 13,710 years. From these considerations it seems certain that the mass of the planetary nebula N. G. C. 7009 is several times that of the Sun. The nebula is therefore competent, from the point of view of its mass, to develop into a system more pretentious than is our solar system.

A few speculations concerning this nebula may not be without interest and value.

The faint extensions to the east and to the west of the elliptical fig-

ure suggest an encircling ring of materials whose principal plane, passing through the nucleus, passes also near our (the observers') position in space. These extensions terminate in condensed nuclei at equal distances from the nucleus and on exactly opposite sides of the nucleus. The faint extensions and condensations may be and probably are largely the effect of the edge-wise projection of such a ring, as in the case of Saturn's rings when the observer is in the plane of the rings. The forms of the two terminating condensations, and especially the wing extending up and out from the east condensation, suggest that we are not precisely in the plane of the assumed ring.



PLANETARY NEBULA N. G. C. 7009 (COMPOSITE DRAWING, FROM CURTIS'S PHOTOGRAPHS OF THE NEBULA MADE WITH THE CROSSLEY REFLECTING TELESCOPE. THE SCALE IS IN SECONDS OF ARC)

The form of the main nebula appears to be ellipsoidal and not chiefly elliptical.

The space immediately surrounding the central nucleus appears to be relatively vacuous. Aside from the nucleus, the principal mass of visible nebulosity exists in the brilliant ring, roughly elliptical as to its inner and outer boundaries, which occupies the region about midway between the nucleus and the outer edge of the nebular structure. The brilliant ring is probably in reality an ellipsoidal shell: the projection of such a shell upon a plane at right angles to the line of sight would nat-

urally show a relatively dark central area, but the projection principle may not be the only one involved.

If this nebula is in process of development into a solar system, the indications are for a system having certain resemblances to our solar system. Our four outer planets have a combined mass 225 times as great as that of the four inner planets. Similarly in N. G. C. 7009, there is apparently a paucity of material to form planets near the nucleus and an abundance of material for planets at greater distances from the nucleus.

In the course of our determinations of nebular velocities in the past three years we have noted a relatively large number of ring forms among the planetaries, and we have observed no extremely-elongated or highly-elliptic gaseous nebulae, and no such forms of gaseous nebulae as would result from thin rings seen edge-wise. According to the probabilities of the case, if the apparent ring nebulae are really of ring form in space, we should have expected to see a number of very elongated elliptic rings and a certain number of relatively long and narrow nebulae. We are accordingly led to question whether the ring nebulae are true rings or whether they are in reality ellipsoidal shells of nebulosity, which appear to be rings wholly or in part by virtue of their projection upon the plane of sight.

Students of cometary orbits in the past five years have apparently established the Kantian view, that comets are bona fide members of our solar system. The observed fact that comets approach the Sun without marked preference for any direction of origin has seemed to be difficult to account for on the more prominent hypotheses of the evolution of the solar system. If the planetary nebula N. G. C. 7009 and other planetaries which involve the ring form have in reality three principal dimensions and are developing into solar systems, the more or less uniform distribution of cometary matter in space of three dimensions does not seem to call for surprise.

A SHORT PERIOD CEPHEID WITH VARIABLE SPECTRUM

By Harlow Shapley

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The most suggestive indication that the stars actually develop from one physical condition represented by a definite spectral type into other closely allied conditions, if it is not in fact the only direct observational evidence of stellar evolution, is afforded by the spectra of certain short

period variable stars. From one star to the next in all regions of the sky we observe continuous gradations of many constant characters, such as spectrum, intrinsic luminosity, and motion. That a star begins, passes through and ends its light-emitting career without change of spectral type is considered impossible. The universal and presumably justifiable assumption is that a given spectrum has developed by imperceptible degrees from an earlier type, and will, given time enough, develop into a later one—that every star, in the course of its life history as a luminous object, passes through many if not most of the principal types and sub-types of the spectral series. This is indeed more than an ordinary assumption; it is a scientific creed. It is now generally accepted by astronomers as an inevitable fact, so nearly axiomatic that its demonstration is no more essential than the proof of the fact of organic evolution. The essential studies concern rather the methods of stellar evolution and its numerous details.

That stellar evolution, as reflected in progressive changes of spectral type, has not been witnessed (the phenomena of novae are nebular rather than stellar) is, of course, no cause to doubt the basic assumption. Observations necessarily have been superficial, and time has been short. It is of interest, therefore, and probably of importance as a proof of stellar evolution to have observed a remarkable change in a star, not only of its light, as in the ordinary variable star, nor merely of its velocity, as in the ordinary double star, but of its very physical make-up. And it is also of importance to know that, when the physical state of a star changes and its intrinsic brightness and surface temperature are affected, the spectrum varies (from one recognizable type to another through known intermediate stages) in exactly the same way as we presume the spectra of all stars have changed or eventually will change. It should be noted, however, that differences in spectra for the most part simply represent dissimilar conditions in the relatively thin atmospheric layer that is the source of the absorption lines; indirectly the deeper structure, even to the center of the star, may be and probably is involved.

In the case under discussion in the present note the variation of spectrum is oscillatory in a period of a few hours, and does not represent, therefore, a secular, irreversible variation of spectral type. It does represent, however, an enormous change in the physical condition at the stellar surface, and exhibits the almost incredible feat of accomplishing in less than two hours time a change of spectrum that for the ordinary development of the ordinary star has been supposed to occupy unknown thousands or millions of years.

The variability of the light of the seventh magnitude star RR Lyrae, the brightest of the cluster-type sub-class of the Cepheid variables, was discovered at Harvard seventeen years ago by the unique method of setting a special automatic photographic telescope to follow the star throughout the night and make a continuous and permanent record of its light. Due to the brightness of the variable and its importance in connection with the study of stellar variation, it has received wide-

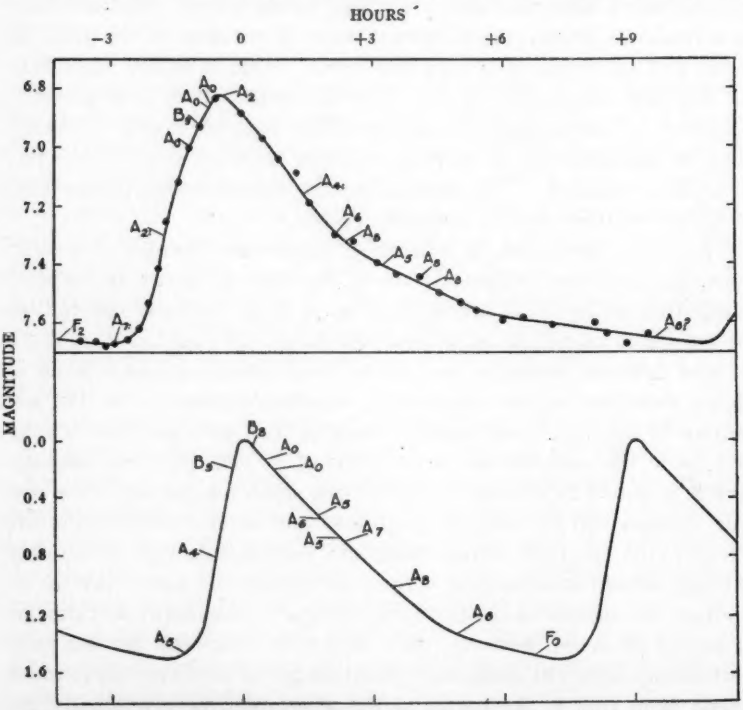

FIG. 1. TOP: MEAN VISUAL LIGHT-CURVE OF RR LYRAE (OBSERVATIONS BY WENDELL) AND VARIATION OF SPECTRUM (SHAPLEY).
BOTTOM: PHOTOGRAPHIC LIGHT-CURVE OF RS BOÖTIS (SEARES AND SHAPLEY) AND VARIATION OF SPECTRUM (PEASE).

spread attention. The light variations have been observed at Harvard, Princeton, the Lick Observatory, the University of Missouri, and at observatories in Italy, Germany, Sweden, and Ireland. Special measures of its position have been made in Finland and in Germany, and observations of the spectrum at Harvard, Lick, and Mount Wilson.

Kiess at the Lick Observatory found the star to be a radial-velocity variable—a spectroscopic binary of the peculiar Cepheid type. This

linked together the cluster-type and the similar longer-period Cepheid variables, all of which show periodic shifts of the spectral lines, permitting, on the Doppler principle, of interpretation as the result of orbital motion in abnormal but improbable double-star systems. The recent work on the spectrum at Mount Wilson with a 10-inch photographic triplet and an objective prism shows that the spectrum varies between classes A and F in the same period as the variation of light and radial velocity.

The star is, then, a periodic variable in at least three different senses. Every thirteen and a half hours, due to some cause as yet not fully understood, the light increases rapidly, more than doubling its intensity in less than two hours. During the same interval the apparent velocity of the star toward the earth increases from 47 to 91 kilometers a second, and the spectrum changes from type F to type A.

This last variation is probably the most remarkable of all, for it implies, on the basis of our present knowledge of stellar radiation, a change in the temperature at the radiating surface of some three thousand degrees centigrade. Presumably the mean color of the light of RR Lyrae changes perceptibly during these cataclysmic two hours, but the evidence on that point is not yet conclusive.

After the maximum of velocity, of light, and of spectrum have been reached, a reaction sets in, sharply at first, then more gradually, all factors decreasing for nearly half a day. The average time between outbursts is known to a fraction of a second, and the hour of a maximum a century from now could be predicted with considerable certainty. Doubtless a common cause underlies all these variations, and their proper interpretation and correlation will constitute the solution of Cepheid variation.

Though variation of spectral type is suspected in the case of a few other cluster-type variables, and in fact, on the basis of the present results, may be suspected with just cause of all short period Cepheids, the observations have definitely proved the spectrum variable for only two stars. For both of these the spectra have been studied at Mount Wilson, the data for RS Boötis being derived from spectrograms made by Pease at the principal focus of the 60-inch reflector. The results for the two variables are shown graphically in the accompanying figure, where ordinates designate brightness in magnitudes and abscissae give time in hours. The light curve of RS Boötis, along which are plotted the determinations of the spectrum class for the corresponding phases of the light variation, is derived from unpublished photographic observations by Mr. Seares and the writer. The light curve of RR Lyrae is derived from observations made at Harvard.

Although the mean period of RR Lyrae is remarkably constant, conspicuous transitory irregularities are present both in the period and in the form of the light curve, much resembling those of the cluster-type variable XX Cygni previously discussed in these PROCEEDINGS. In particular, there is a slow but definite variation in the length of the period which seems to complete its cycle in about sixteen years; and an oscillation in the time of the rise to maximum light which at times appears to have a uniform period of about forty days, and at other times to be erratic in both period and amplitude. The formula:

$$\text{Max.} = 2414856.451 + 0^d.566831\text{E} - 0^d.024 \sin (0°.0340\text{E} - 104°.5) + [0^d.004 + 0^d.013 \cos \frac{2\pi}{70} (\text{E} - 45)]$$

in which E represents the number of periods, predicts the individual maxima with high accuracy for the first seven of eight hundred periods after the initial epoch, and by dropping the short period harmonic it gives the mean maxima accurately for at least fifteen years from the initial date. The complete discussion of these features, based on all the available series of light measures and on the recent observations of the spectrum, will be published as a *Contribution from the Mount Wilson Solar Observatory*.

THE SPECTRUM OF δ CEPHEI

By Walter S. Adams and Harlow Shapley

MOUNT WILSON SOLAR OBSERVATORY, CARNEGIE INSTITUTION OF WASHINGTON

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About 130 years ago Goodricke discovered the periodic fluctuations in the light of the naked-eye star δ Cephei—the eighth variable known to astronomers. None of those previously recorded exhibited the type of light variation peculiar to this fourth magnitude star, but of the four or five thousand variables found since that time more than half show the same character of light fluctuations. The class has, accordingly, received the appropriate designation of Cepheid variables. A century after the discovery of the light variations, δ Cephei was also found to vary in radial velocity—the sixth spectroscopic variable to be placed on record. Again the variation differed from that of other stars. The period of the oscillation of the spectral lines is identical with that of the light variation, but the orbital elements, derived on the hypothesis that the star is a binary, permit no satisfactory explanation of the light variations. When other velocity-variables of this character were found

they were classed as Cepheids, and, in fact, every light-variable of the Cepheid type, when put to the test, has proved to be a velocity-variable with the distinct Cepheid peculiarities. Any definite contribution to the explanation of the light or velocity variations of a single Cepheid, therefore, is of unusual value in that it involves the interpretation of the majority of all variable stars.

The present state of our knowledge of the causes of Cepheid variation has been discussed in former papers. The clearest outcome of recent studies of the subject is the apparent impossibility of relating the velocity variations to orbital motions. Probably the strongest evidence on this point is the synchronous variations of the magnitude and the spectral class, recently observed at Mount Wilson for two cluster-type Cepheids. Such variations of spectrum have received little consideration heretofore, notwithstanding the known change in color from maximum to minimum light that would seemingly demand the changes in spectral lines that distinguish one class of spectrum from another.

The question naturally arises whether in suitable investigations of the brightest and most typical Cepheids changes of spectrum would not be found analogous to those observed in the less typical variables RS Boötis and RR Lyrae. Might it not also be possible with a special analysis of high dispersion plates to bring out other characteristics of the spectrum that would throw light on the causes underlying the light and velocity variations? With these points in view an investigation of the spectrum of δ Cephei was made, and the results are briefly outlined in the following paragraphs. The discovery of the conspicuous change of the spectral class of the first and best known Cepheid indicates that constancy of spectrum is to be expected for none of this class of variables, and, coupled with the remarkable behavior of the spectral lines, suggests that the possibility of a completely satisfactory theory of Cepheid variation is not necessarily remote.

Two photographs of the spectrum of δ Cephei were obtained on the nights of December 23 and 24, 1915, with the Cassegrain spectrograph and 60-inch reflector. The full optical train of three prisms and a 102 cm. camera was employed. This combination gives a linear scale at $H\gamma$ of 5.3 angstroms to the millimeter. The data for the photographs are as follows:

Plate γ 4571	Dec. 23, G.M.T. 15^h4^m	Exposure time 225^m
Plate γ 4578	Dec. 24, G.M.T. 14^h7^m	Exposure time 120^m

The first photograph was taken under exceptionally poor conditions, and this fact is responsible for the extended exposure time. As it is,

the plate, though well measurable, is somewhat under-exposed. Especial care was taken in adjusting the spectrograph to secure good definition in the region of the hydrogen line $H\gamma$. As a result the spectrum lines are very sharply defined from λ 4200 to λ 4500, but begin to show diffuseness to the red of this point. They are, however, measurable as far as λ 4600.

A comparison of the two negatives under a Hartmann spectrocomparator at once showed some important differences beween the two spectra.[1] These may be summarized as follows. The photograph of

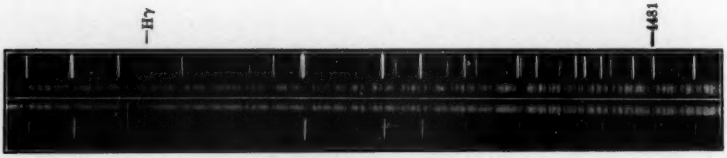

FIG. 1. SPECTRA OF δ CEPHEI
Top: Spectrum near Minimum of Light
Bottom: Spectrum near Maximum of Light

December 24 was taken near the star's maximum of light, and that of December 23 not far from minimum.

	γ 4578 (*maximum*)	γ 4571 (*minimum*)
Hydrogen line $H\gamma$	Strong	Much weakened
Enchanced lines of *Fe*, *Ti*, *Sr*, and *Cr*	Strong	Much weakened
λ 4481, enhanced *Mg*	Very strong	Much weakened
λ 4227 of calcium	Strong	Strengthened
Low temperature lines of *Ca*, *Fe*, *Ti*, and *Cr*	Weak	Strengthened
Continuous spectrum	Strong in violet	Weakened in violet

The observation on the continuous spectrum is made somewhat uncertain by the general under-exposure of the plate taken near minimum, but the result is apparently as given. The lines on this photograph appear to be broader and less sharply defined than those on the plate at maximum, and this effect we consider as probably genuine after all necessary allowance has been made for the difference in the quality of the two negatives. There is, however, no evidence of lack of symmetry in the spectrum lines either at maximum or minimum, nor of the presence of a secondary spectrum. It appears from these observations that the changes follow a definite tendency. At maximum the high temperature lines are very strong, and the low temperature lines are weak; while at minimum the reverse is the case. The conclusion appears to be justified that the temperature of the gases constituting the star's absorbing envelope is higher at maximum of light than at minimum.

A method of applying the Harvard system of classification to the determination of spectral type through the aid of numerical relationships between the intensities of the hydrogen lines and certain other selected lines will be described by one of us in another Communication. The use of this method gives in the case of δ Cephei:

At maximum F4, At minimum G2

The variation in type, accordingly, amounts to 8 divisions of the Harvard scale.

An independent determination of the spectrum and its change can be made from the comparison of the mean visual and photographic light curves. Nearly twenty visual curves have been made during the last hundred years, but only the one by Stebbins is based on measures with a photometer. The only photographic curve is due to Wirtz. It is possible to reduce both curves to the international magnitude scale; the difference between them for any phase gives the color index, which may be transformed into spectral type by known relations. Hence we derive in stellar magnitudes:

$$\text{Visual Range} = 4.25 - 3.49 = 0.76,$$
$$\text{Photographic Range} = 5.15 - 3.90 = 1.25;$$

and for the spectral class at light maximum F2, at minimum G4. The time of plate γ 4571 coincides with velocity minimum but succeeds the minima of the light and color curves by several hours. Allowing for this we derive from the color curve, for comparison with the direct classification above, the satisfactory result:

At maximum F2, At minimum G0

An important conclusion is that all genuine color variations observed in Cepheids may be directly interpreted as normal changes in spectral class.

Associated with the variation in spectral type in δ Cephei is a variation in the intensity of certain spectrum lines which have been found to fluctuate with the intrinsic luminosities of the stars in which they occur. The use of the intensities of these lines in calculations of absolute stellar magnitudes will be described elsewhere. The application of the method to the case of δ Cephei is of considerable interest because of the accurately known range of variation in apparent, and hence, of course, in absolute magnitude. Unfortunately the spectrum of the star at maximum (F4) is of a type not well suited for the use of the method and the results are necessarily approximate. The variation in absolute magnitude found in this way, however, is entitled to considerably more weight than are

the magnitude values themselves. The two photographs give the following values:

Minimum M = +1.8, Maximum M = +0.7, Variation 1.1

The two conclusions to be drawn from these results are: First, that they agree with results based on other considerations such as proper motion in indicating that δ Cephei is a star of very high intrinsic brightness; second, the variation in magnitude derived from characteristics of the spectrum, 1.1, is in very fair accord with the value 0.8, the star's known range of visual magnitude.

In selecting the lines to be measured for the determination of radial velocity from the two photographs the consideration was borne in mind that different lines might give different velocities. Accordingly, an extended list was made out including: (1) a large number of enhanced lines; (2) iron lines which show a wide variety of displacements under pressure; (3) special lines such as $H\gamma$ and λ 4227 of calcium. The photograph taken near maximum was measured by Mrs. Monk and Adams, the result given being the mean for the two observers; the other plate was measured by Mrs. Monk alone:

	No. of Lines	*Radial Velocity*
γ 4571 (minimum)	55	+ 3.8 km/sec
γ 4578 (maximum)	68	−35.2

The radial velocity for the times of the Mount Wilson photographs can be predicted by means of the elements of the velocity variation, derived by Moore[2] from Lick Observatory spectrograms, using in this computation the improved light elements by Luizet.[3] The results are:

Minimum, +3.7 km/sec, Maximum, −35.2.

The exact agreement between the observed and computed velocities not only checks the results from the two observatories and indicates that the range of velocity variation is the same now as eight years ago, but also supports Moore's conclusion that the velocity of the center of mass does not undergo the variation assigned it by Belopolsky.

A comparison of the velocities given by the individual lines on these photographs shows some interesting features. Of these the most important is the behavior of the iron lines which in the laboratory show small displacements under pressure as compared with those which show relatively large shifts. The results for the photograph taken near the star's maximum are shown in detail in Table I. The pressure shifts in angstrom units per atmosphere, given under Δ, are taken from unpublished laboratory results by Gale and Adams. The velocities, v, are in kilometers, and are the measured values, not corrected for the earth's motion.

The mean value of the velocity as derived from the lines of large pressure shift is 1.58 km greater than that from those of small shift, and it is noteworthy that not one of the velocities of the lines in the latter group reaches the mean value for the large shift lines. Similarly only two of these reach values as low as the mean for the lines of small pressure displacement.

The photograph taken near the star's minimum shows similar results, although it is entitled to considerably less weight because of its quality and the fact that but a single measurement has been made upon it.

TABLE I

Small pressure shift

LINE	Δ A	v KM	LINE	Δ A	v KM
4250.9	+0.001	−20.1	4415.2	+0.004	−20.4
4271.9	0.001	18.9	4422.7	0.001	21.2
4325.9	0.002	21.2	4466.7	0.003	19.5
4352.9	0.002	19.6	4442.5	0.005	19.6
4376.1	0.002	18.3	4482.3	0.003	20.1
4383.7	0.003	20.0	4531.3	0.003	18.3
4404.9	0.004	19.9			
Mean.....				+0.0027	−19.79

Large pressure shift

LINE	Δ A	v KM	LINE	Δ A	v KM
4222.3	+0.007	−20.8	4271.3	+0.008	−22.6
4227.6	0.017	24.0	4407.8	0.012	20.6
4233.7	0.006	20.3	4430.7	0.007	19.6
4236.1	0.007	23.7	4447.8	0.006	19.4
4238.9	0.012	22.5	4494.7	0.007	20.5
4250.2	0.011	21.5	4528.7	0.006	21.7
4260.6	0.010	20.5			
Mean.....				+0.0090	−21.37

Combining the results with weights of two and one we obtain the following value for the difference in velocity:

Large Δ lines (+0.009 A) − Small Δ lines (+0.003 A) = −1.22 km

The sign of this difference is opposite to that which would be found if pressure is the agent producing it and if the effective pressure at the star's surface where these lines are formed is greater than one atmosphere. It is, however, the sign which would be found if the stellar pressure is less than one atmosphere. In that case the iron comparison lines on the negatives, which of course are taken at atmospheric pressure,

would be shifted toward longer wave-lengths with reference to the stellar lines by amounts corresponding to the observed pressure displacements. It seems to us probable that this is the explanation of at least the principal part of the observed difference, although differential radial currents in the star's atmosphere may account for a portion of it.

An important confirmation of this result is furnished by Mr. St. John's measurements of the displacements of solar lines relative to the arc spectrum lines of iron. He has kindly furnished us with the provisional values obtained for the lines given in Table I, and from these we find: first, that the small pressure shift lines are displaced toward the red in the sun relative to the arc lines by an average amount of 0.009 angstrom; second, that the large pressure shift lines are displaced toward the violet 0.004 angstrom. These differences are in the same direction as in the case of δ Cephei and the absolute amount is of much the same order. Thus we find:

Sun	Large Δ lines $-$ Small Δ lines	$=$	-0.013 A	$=$ -0.9 km
δ Cephei	" "			$=$ -1.2 km

It is well known that the pressures in the upper portions of the sun's atmosphere where the stronger lines of the spectrum appear to find their origin are extremely low.

A comparison of the velocities given by the enhanced lines in δ Cephei with those given by such as are not enhanced indicates a distinct shift of the former toward longer wave-lengths. In the case of the photograph taken near maximum the difference amounts to $+0.86$ km, or about $+0.012$ angstrom. The other photograph gives a similar value of larger amount. A difference of the same character was found by Adams some years ago for the enhanced lines in the spectra of Sirius and Procyon.[4] The values were $+0.90$ km for Sirius and $+0.58$ for Procyon, with which the result found for δ Cephei is in very fair agreement. It is probable that the explanation of these results is to be found mainly in radial convection currents in the stellar atmospheres.

[1] Belopolsky and Lohmann at Pulkova have described some differences in the intensities of the lines at maximum and minimum light, but the dispersion employed makes it impossible to identify the lines with certainty.

[2] Moore, *Lick Obs. Bull.*, 7, 153 (1913).

[3] Luizet, *Ann. l'Univ. Lyon*, N. S., 33, 42 (1912).

[4] *Astrophys. J.*, 33, (1911).

INVESTIGATIONS IN STELLAR SPECTROSCOPY. I. A QUANTITATIVE METHOD OF CLASSIFYING STELLAR SPECTRA

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The basis of the classification of stellar spectra is at present largely empirical. In the absence of sufficient knowledge as to the modifications of spectra produced by different physical conditions it has not been possible to establish with certainty a system of classification which will represent the actual order of stellar development. Hence the stars have been classified into types simply in accordance with the characteristics of their spectra. The appearance of new lines and the disappearance of others, systematic variations in the intensities of certain lines, the presence of bands, the intensity of the continuous spectrum, and other similar criteria have been used to separate the stars into several spectral groups.

To some extent the system of classification now in general use by astronomers, that devised by the Harvard Observatory, probably has a physical basis. Thus it is well known that the differences between the spectrum of the sun and that of a star like Arcturus are very similar to those between the spectrum of the sun and that of sun-spots. In the latter case investigations have shown that a reduction of temperature is the principal agent in producing the modifications observed. Similarly the presence of bands characteristic of certain compounds which are found in the spectra of stars like α Orionis is an indication of relatively low temperature. Accordingly it seems probable that the successive types of stellar spectra, represented by the sun, Arcturus, and α Orionis, are characterized by successively lower temperatures in the gases forming the atmospheres of these stars. This does not of necessity indicate, however, that Arcturus and α Orionis have developed from stars like our sun. Lockyer and some others consider that the curve of stellar development has both an ascending and a descending branch, and that some stars of low temperature will become hotter before beginning to cool permanently. Stars which differ greatly in size and mass must almost certainly differ in the rate, and quite possibly in the order, of their development as well.

The principal lines used in the Harvard system of classification for the separation of stars into the several types are certain lines due to calcium, the more prominent lines of such metals as iron, and, most im-

portant of all, the hydrogen lines. In accordance with this system the stars are divided into seven main types designated by the letters B, A, F, G, K, M, and N, with intermediate types indicated on a scale extending from zero to ten. Thus G5 indicates a type halfway between types G and K. The B stars are characterized by helium and hydrogen absorption lines. In the A stars the helium lines disappear, the hydrogen lines reach their maximum intensity, and faint metallic lines begin to appear. These lines grow stronger and the hydrogen lines weaker in the successive types F, G, and K, the low temperature lines in particular increasing rapidly in intensity between the G and K types. The sun is a typical G0 star. The M and N stars are distinguished by the presence of bands, in the one case of a compound of titanium, and in the other of carbon.

The Harvard system of classification in general meets the requirements of spectral observations in a most excellent way. There is, however, in published descriptions of its application a serious lack of numerical relationships between the intensities of the lines compared, and as a result a considerable uncertainty arises in the determination of spectral types. Since in many astronomical investigations a comparison is instituted between stars of very closely the same type it is important to reduce the classification of stellar spectra to as accurate a basis as possible. The following brief description of the method employed at Mount Wilson is given for two purposes: first, because it replaces to a considerable extent direct estimations of spectral type by numerical estimates of relative line intensity which may be made with much higher accuracy; and second, because the method provides the material upon which several investigations have been based. It was devised in large measure by Dr. Kohlschütter, and has been used with but slight modifications since his departure from Mount Wilson.

The material available for classification purposes consists of several thousand photographs of stellar spectra taken with a one prism slit spectrograph and the sixty-inch reflector. About two-thirds of these spectra are of types succeeding F0. On most of the photographs the region of spectrum in best definition extends from λ 4200 to λ 4900. It includes, therefore the two hydrogen lines $H\gamma$ and $H\beta$, the important calcium line at λ 4227, and some of the most prominent iron lines in the entire spectrum. Since the hydrogen lines show a rapid decrease in intensity with the successive types F, G, K and M, and form by far the most important criterion in the derivation of spectral type, accurate determinations of their intensity relative to other lines in the spectrum are essential. Accordingly several adjacent iron lines have been selected

which show but a moderate change of intensity in these types, and estimates are made on an arbitrary scale, extending from zero to ten, of the differences in intensity between the hydrogen lines and this selected list. The calcium line λ 4227 is also compared with $H\gamma$ in the types F0 to G5, beyond G5 the differences becoming too great to provide satisfactory determinations. The list of pairs of lines finally adopted for classification purposes is given in Table I.

TABLE I

Lines Compared	*Range of Type*
$H\gamma$ and λ 4227, *Ca*	F0 to G5
$H\gamma$ " λ 4326, *Fe*	F3 to Ma
$H\gamma$ " λ 4352, *Fe, Mg*	F0 to Ma
$H\gamma$ " λ 4383, *Fe*	F0 to G5
$H\gamma$ " λ 4405, *Fe*	F3 to Ma
$H\beta$ " λ 4872, *Fe*	G0 to Ma
$H\beta$ " λ 4957, *Fe*	G0 to Ma

The scale of classification was adapted to the Harvard system by selecting a considerable number of stars for which Harvard determinations were available, and making estimates of the relative intensities of these pairs of lines in the stars selected. The values were then plotted against the average types of these stars, and smooth curves were drawn through the several points. These curves provide the means of converting determinations of relative line intensity into determinations of spectral type. The curves are shown in figure 1. For reasons which will appear later, they are based upon stars of large proper motion alone, and the material may, therefore, be regarded as homogeneous in character.

To illustrate the use of these curves I have selected as examples the stars Groom. 3357, Piazzi 0^h130, Groom. 145 and Lal. 19022. The estimated differences of intensity for these stars, as determined from three photographs of their spectra, are given in Table II.

TABLE II

	$\frac{4226}{H\gamma}$	$\frac{4326}{H\gamma}$	$\frac{H\gamma}{4352}$	$\frac{H\gamma}{4383}$	$\frac{H\gamma}{4405}$	$\frac{H\beta}{4872}$	$\frac{H\beta}{4957}$	*Bands*
Groom. 3357	0.0	−5.3	+7.0	+4.3	+7.0			
Pi 0^h130		+3.7	+0.7		−1.3	+3.3	+1.0	
Groom. 145		+5.3	−2.0		−4.0	−0.2	−1.8	
Lal. 19022		+6.3	−3.3		−5.0	−1.3	−3.3	1

With the aid of tables constructed from the curves we obtain the following determinations of spectral type from the separate pairs of lines:

		Mean	*Probable Error*
Groom. 3357	F7 F7 F4 F6 F5	F6	±1.0
Pi 0^h130	G7 G5 G6 G4 G5	G5	0.8
Groom. 145	K1 K1 K3 K3 K2	K2	0.6
Lal. 19022	K7 K7 Ma K6 K6	K7	1.6

The average probable error of the determination of type for these four stars is ± 1.0, and this is about the value obtained for several hundred stars classified in this way. It is evident that the accuracy will be

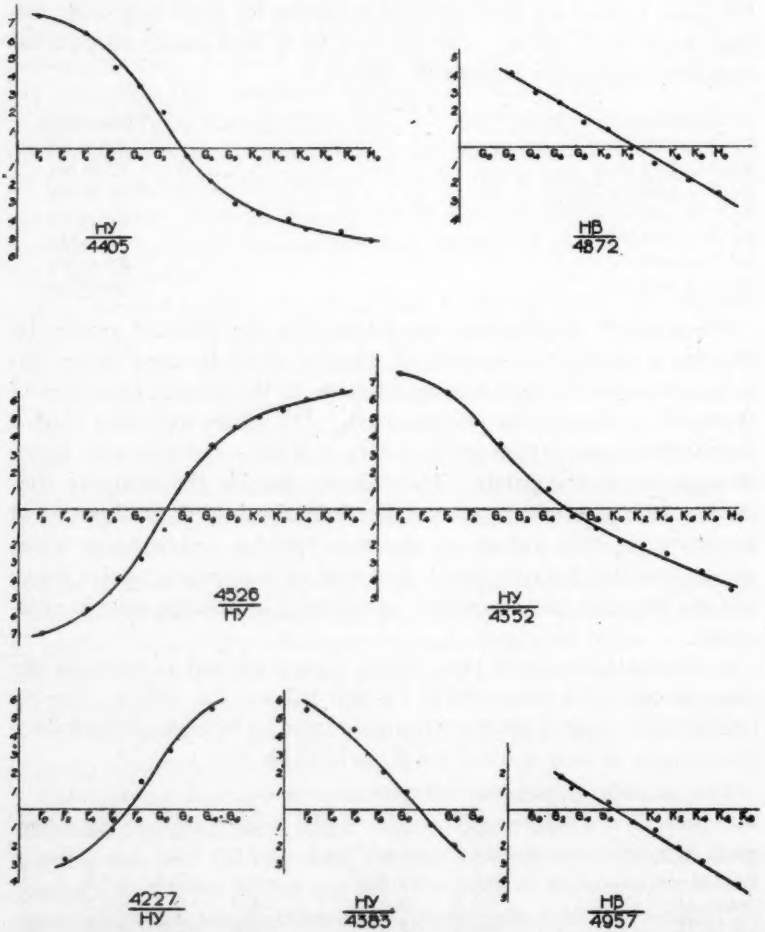

FIG. 1. CURVES USED IN STELLAR CLASSIFICATION SHOWING THE VARIATION OF THE RELATIVE INTENSITIES OF PAIRS OF LINES WITH SPECTRAL TYPE. THE ORDINATES ARE THE DIFFERENCES OF RELATIVE INTENSITY.

least when the lines compared differ greatly in intensity, as in the types F0–F9 and K5–Ma, and greatest when the lines are of nearly equal intensity.

This simple method of classification may be recommended as being rapid of operation, and free from the difficulties connected with the comparison of separate photographs with one another. It requires the establishment of a scale of relative-intensity-estimates by the observer, but this is a very simple matter when the range employed is small. To some extent the scale will be dependent upon the dispersion of the spectrograph employed since several of the lines used are compound in character. With the single prism spectrograph at Mount Wilson the same reduction curves have been used successfully for photographs on which the linear dispersion varies from 16 to 90 angstrom units to the millimeter at the center of the spectrum.

In connection with the classification of stellar spectra a number of photographs have been made with a Koch microphotometer of the intensity curves of some of the pairs of lines employed in the comparison. There are numerous practical difficulties connected with the use of this instrument for lines as narrow and as short as those in stellar spectra, and it is doubtful whether the accuracy obtained is of so high an order as to justify the use of so laborious a method for stellar classification. It is probable, however, that it might be used to advantage in the selection of standard stars of reference in which a knowledge of the absolute intensities of certain spectrum lines would be of great value.

INVESTIGATIONS IN STELLAR SPECTROSCOPY. II. A SPECTROSCOPIC METHOD OF DETERMINING STELLAR PARALLAXES

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The question whether the intrinsic brightness of a star may not have an appreciable effect upon its spectrum is one with important applications in astronomy. If two stars which have closely the same type of spectrum differ very greatly in luminosity it is probable that they also differ greatly in size, mass, and in the depth of the atmospheres surrounding them Accordingly we might hope to find in these stars certain variations in the intensity and character of such spectrum lines as are peculiarly sensitive to the physical conditions of the gases in which they find their origin, in spite of the close correspondence of the two spectra in general. If such variations exist and a relationship may be derived between the intensities of these lines and the intrinsic brightness

of the stars in which they occur, we have available a means of determining the absolute magnitudes* of stars, and hence their distances.

The first attempt to detect such lines was made by Hertzsprung, who concluded that the strontium line at λ 4077 gave some indication of varying with the absolute magnitudes of the stars in whose spectra it appeared. Quite independently Dr. Kohlschütter in the course of his studies of the classification of the Mount Wilson stellar spectra found two or three lines which appeared to vary in this way, and some results of an application of these lines to the determination of absolute manitudes were published in 1914. Since that time the work has been extended greatly with the aid of the additional material available. The results of the investigation and of an attempt to utilize these criteria for the derivation of stellar distances are contained in this communication.

The first essential in beginning this research was an accurate classification of the stellar spectra into the several types. This was carried out by the method already described (These PROCEEDINGS, 1, 481). Stars of the same type of spectrum but of very different absolute brightness were then compared with one another, and the relative intensities of the different spectral lines were examined carefully.

To illustrate the procedure we may take as an example the two stars 61^1 Cygni and α Tauri. The parallaxes of these stars are 0.″31 and 0.″07, respectively, and their apparent magnitudes are 5.6 and 1.1. Their absolute magnitudes may be computed from the equation

$$M = m + 5 + 5 \log \pi$$

in which M is the absolute magnitude, m the apparent magnitude, and π the parallax. The absolute magnitudes, accordingly, are 8.0 and 0.4; that is, the luminosity of α Tauri is over 1100 times as great as that of 61^1 Cygni. A comparison of the spectra of the two stars side by side on a Hartmann spectrocomparator shows several points of difference. Of these, two are most important. The calcium line at λ 4455 is very strong in 61^1 Cygni and relatively weak in α Tauri; and the strontium line at λ 4216 is weak in 61^1 Cygni and strong in α Tauri. That this difference in behavior depends upon physical conditions in the stars and is not merely accidental is made almost certain by solar investigations. The line λ 4455 of calcium is greatly strengthened in the spectrum of

*The absolute magnitude of a star is its apparent magnitude when reduced to unit distance. The unit commonly employed is the distance corresponding to a parallax of 0″.1. On this scale the absolute magnitude of the sun would be 5.5, or 4.8, if more recent, and probably better, values of the sun's photometric brightness are employed.

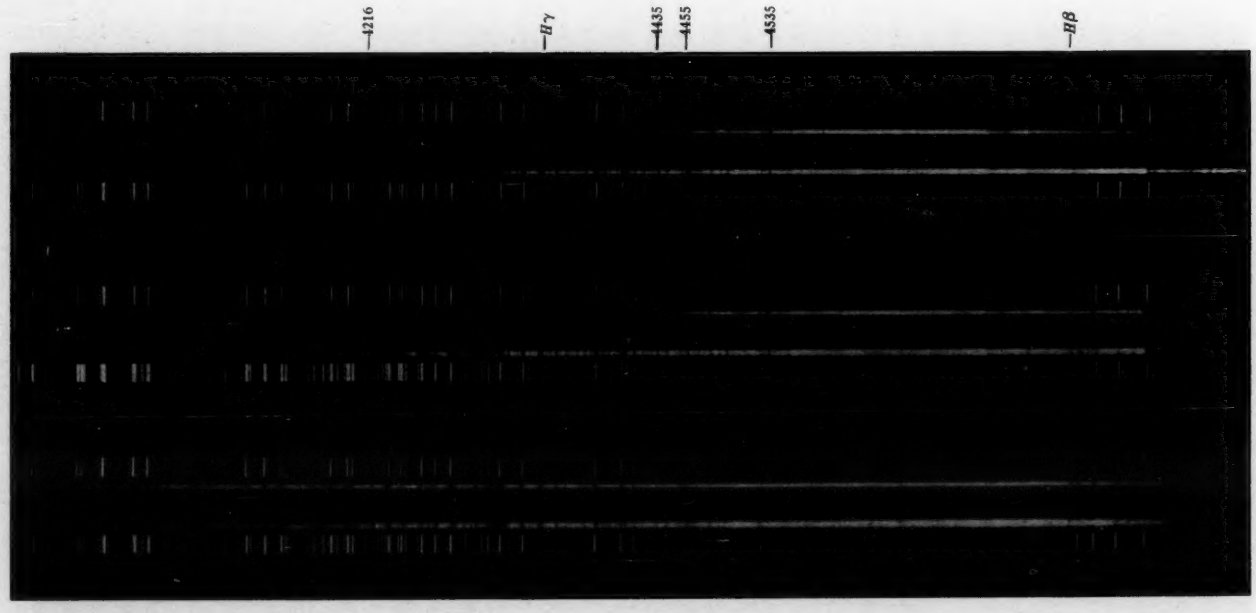

FIG. 1. SPECTRA OF PAIRS OF STARS OF DIFFERENT ABSOLUTE MAGNITUDE SHOWING VARIATION OF THE INTENSITY OF CERTAIN LINES WITH MAGNITUDE.

(a) Groombridge 34; (b) Boss 1560. (c) Lalande 21185; (d) Boss 2921. (e) Piazzi 5^h 146; (f) Boss 1513.

sun-spots, and increases in intensity with reduction in temperature. The line λ 4216 of strontium, on the other hand, is an enhanced line, that is stronger in the spectrum of the spark than of the arc, and is probably a high temperature line. It is very prominent in the spectrum of the sun's limb when photographed at eclipses, and also in the upper chromosphere. Numerous other smaller differences between the spectra of α Tauri and 61[1] Cygni all point in the same direction; the low temperature lines strengthened in sun-spots are stronger in 61[1] Cygni; the enhanced lines are stronger in α Tauri.

It has seemed preferable, however, for two reasons to use only these two lines in the absolute magnitude investigation. First, because they show the effect most markedly; and second, because they appear to be influenced but slightly by closely adjoining lines which blend with them. Among other lines which show the effect plainly, reference should be made to λ 4435 of calcium and λ 4535 of titanium, which are strong in intrinsically faint stars, and to two lines at λ 4395 and λ 4408 which are strong in the brighter stars. The line at λ 4395 is probably due to enhanced titanium. As will appear later, in the course of a discussion of M type stars, the hydrogen lines themselves seem to vary with absolute magnitude, at least in certain types of spectra. This should prove of fundamental importance in further investigations of stellar luminosity.

After the behavior of the two lines λ 4216 and λ 4455 had been examined in a large number of stars, and the systematic differences had been found to persist through a wide range of spectral type, the attempt was made to establish a numerical relationship between the intensities of these lines and the absolute magnitudes of the stars in which they occur. As in the case of the hydrogen lines used for classification purposes, lines were selected near λ 4216 and λ 4455, with which the intensities of these lines were compared, the differences of intensity being estimated on a scale extending from zero to ten. The pairs of lines finally adopted for all of this work are as follows:

(a) λ 4216, *Sr* and λ 4250, *Fe*
(b) λ 4455, *Ca* λ 4462, *Fe*, *Mn*
(c) λ 4455, *Ca* λ 4495, *Fe*

For convenience of reference these pairs of lines will be designated in the future as (a), (b) and (c). The value (a) = −2, for example, denotes that λ 4216 is estimated to be two units fainter than λ 4250.

As soon as the estimates had been completed a number of the stars with well-determined parallaxes were selected, their absolute magnitudes were computed, and curves were constructed in which the observed differences of intensity for each pair of lines formed the abscissae, and the

absolute magnitudes the ordinates. The stars were divided into five groups according to spectral type and curves were drawn for each group. The groups are as follows:

F0–F6; F7–G7; G8–K4; K5–K9; M

The curves are so nearly straight lines in the case of the first three of these groups that straight lines have been adopted, the constants being derived by least square solutions. In the K5–K9 group the curve for (a) is a straight line but not for (b) or (c). It is probable that there are no straight lines in the M group, but this is very uncertain. The significance of a straight line is, of course, that the intensity of the spectrum line varies uniformly with the absolute magnitude.

The most serious difficulty in the construction of these curves is the scarcity of parallax determinations on stars of high luminosity. Parallax observers have confined their attention almost wholly to stars of large proper motion which promise to yield large parallaxes. With the aid, however, of the Yale observations on the very bright stars, and some most valuable determinations by Mr. van Maanen of the parallaxes of certain stars of small proper motion, a number of stars of very high luminosity were selected upon which the lower portions of the curves could be based. Particularly in the cases of the K5–K9 and the M groups these portions of the curves are still most uncertain, and must be adjusted with the aid of additional parallax observations when they become available.

The list of formulae derived for the several groups is given in Table I. The equations are from my own observations. A similar list, in which the constants differ slightly, has been obtained from the determinations of Miss Burwell, who has carried out a complete series of estimations of the line intensities in these stars. In the formulae, M is the absolute magnitude, and Δ the estimated difference of intensity for each of the pairs of lines.

TABLE I

	(a)	(b)	(c)
F0–F6	$M = -1.9\Delta + 5.2$	$M = +2.0\Delta + 3.3$	$M = +2.8\Delta - 2.6$
F7–G7	$M = -1.4\Delta + 4.9$	$M = +2.0\Delta + 3.3$	$M = +2.8\Delta - 2.6$
G8–K4	$M = -1.6\Delta + 4.7$	$M = +1.6\Delta + 5.1$	$M = +2.3\Delta - 0.3$
K5–K9	$M = -1.8\Delta + 5.0$	Curve	Curve
M (low luminosity)	$M = -1.5\Delta + 6.9$	Curve	Curve

The equation and curves in the case of the M stars are applicable only to the stars of low luminosity. In the case of the F0–F6 stars it is doubtful whether the equations given, which for (b) and (c) are the same as in the G group, are other than rough approximations. The en-

hanced lines in the early F stars are normally so prominent that it is not surprising that the method begins to break down at this point.

To illustrate the use of the formulae and curves we may select as illustrations a few stars of different spectral types and magnitudes. These are collected in Table II. The classification is from Mount Wilson determinations.

TABLE II

STAR	MAG.	TYPE	Δ			M			PARALLAX		
			(a)	(b)	(c)	(a)	(b)	(c)	Mean	Comp.	Obs.
Pi 10^h96....	7.6	F5	−0.7	+0.7	+3.0	+6.5	4.7	5.8	5.7	+0″.04	+0″.04
Sun.....		G0	−0.5	+0.5	+3.0	+5.6	4.3	5.8	5.2		
Lal. 38287..	7.2	G5	−1.8	+1.5	+3.5	+7.4	+6.3	+6.2	7.3	+0.10	+0.09
α Arietis...	2.2	K0	+2.5	−2.4	+0.2	+1.0	1.3	+0.2	0.8	+0.05	+0.09
α Tauri....	1.1	K5	+3.0	−2.0	+0.5	−0.4	+1.9	+0.5	0.7	+0.08	+0.07
61^2 Cygni..	6.3	K8	−1.8	+5.8	+7.7	+8.2	9.3	8.9	8.8	+0.32	+0.31
Groom. 34..	8.2	Ma	−2.2	+6.8	+9.2	+10.2	10.5	10.4	10.4	+0.28	+0.28

The parallaxes are computed from the absolute magnitudes by the formula, to which reference has already been made,

$$5 \log \pi = M - m - 5.$$

The results are given in the next to the last column of the table, and the measured parallaxes in the final column.

INVESTIGATIONS IN STELLAR SPECTROSCOPY. III. APPLICATION OF A SPECTROSCOPIC METHOD OF DETERMINING STELLAR DISTANCES TO STARS OF MEASURED PARALLAX

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A definite test of the value of this method of deriving stellar parallaxes can be made only through a comparison with all available data on measured parallaxes. Since the evidence depends directly on individual values it is necessary for this purpose to present tables of a somewhat extended character.

It is evident that in the case of the stars whose absolute magnitudes, as computed from the measured parallaxes have been used in the derivation of the relationship between line intensity and absolute magnitude, the mean values of the magnitude will necessarily be identical with those derived from the formulae. The agreement of the measured and the

computed parallaxes of the individual stars, however, serves as important evidence bearing on the validity of the method.

In Tables I and II are collected all of the stars with measured parallaxes equal to or exceeding +0".05 for which we have spectral observations. Table I contains the stars used in the derivation of the curves, but in Table II the values are entirely independent, none of these stars having been used previously. This table, accordingly, serves as a most exacting test of the value of this means of computing parallaxes.

TABLE I

STAR	MAG.	TYPE	M		PARALLAX			Measured parallax	Authority
			A	B	A	B	Mean		
F8-G7									
μ Cassiop.....	5.2	G3	+4.7	+5.2	+0".08	+0".10	+0".09	+0".11	K.
ι Persei.....	4.2	G1	3.3	3.3	0.07	0.07	0.07	0.11	K.
Lal. 6888.....	8.4	G7	7.5	7.5	0.07	0.07	0.07	0.06	K.
α Aurigae.....	0.2	G0	0.5	0.6	0.11	0.12	0.12	0.09	Y.
λ Aurigae.....	4.8	G1	4.8	5.0	0.10	0.11	0.11	0.11	K.
Lal. 11196.....	6.5	G1	5.0	6.6	0.05	0.10	0.08	0.09	K.
Lal. 15565.....	6.9	G4	6.3	6.4	0.08	0.08	0.08	0.09	K.
Pi 7^h321.....	7.0	G2	7.0	6.1	0.10	0.07	0.08	0.05	K.
Groom. 1596...	8.2	G6	5.1	5.4	0.02	0.03	0.03	0.07	Y.
20 Leo Min.....	5.6	G4	4.7	5.6	0.07	0.10	0.08	0.07	K.
Groom. 1830...	6.5	G6	5.9	5.9	0.08	0.08	0.08	0.10	K.
Groom. 1855...	7.4	G7	6.0	5.4	0.05	0.04	0.05	0.06	K.
Lal. 22908.....	7.5	G3	5.8	5.8	0.06	0.06	0.06	0.09	Y.
Lal. 30694.....	6.8	G6	5.3	5.9	0.05	0.07	0.06	0.07	K.
Lal. 31132.....	6.7	G4	6.4	7.0	0.09	0.11	0.10	0.14	Y.
72 w Herculis..	5.4	G1	6.3	5.9	0.15	0.13	0.14	0.12	K.
31 b Aquilae...	5.2	G7	5.3	4.8	0.10	0.08	0.09	0.07	K.
Lal. 37120.....	6.6	F9	4.7	5.3	0.04	0.05	0.05	0.06	Y.
Lal. 38287.....	7.2	G5	7.3	6.7	0.10	0.08	0.09	0.09	Y.
85 Pegasi.....	5.9	G1	5.8	6.1	0.10	0.11	0.10	0.09	S. & M.
G8-K4									
54 Piscium....	6.1	K0	+7.0	+6.4	+0".15	+0".11	+0.13	+0".14	Y.
Mayer 20.....	5.8	K3	7.2	7.1	0.19	0.18	0.18	0.16	K.
Pi 1^h159.....	5.7	K0	6.2	4.9	0.13	0.07	0.10	0.10	K.
α Arietis.....	2.2	K0	0.8	−0.1	0.05	0.03	0.04	0.05	K.
W.B. 3^h113....	7.8	K0	6.5	6.3	0.06	0.05	0.06	0.08	Y.
Pi 5^h146.....	6.4	G9	6.2	5.4	06.0	0.06	0.08	0.12	K.
Lal. 13427.....	8.2	K3	7.0	7.2	0.06	0.06	0.06	0.05	K.
β Gemin.....	1.2	G8	−0.1	1.0	0.05	0.09	0.07	0.06	Y.
Lal. 16304.....	6.0	G8	7.4	6.9	0.19	0.15	0.17	0.11	Y., F.
55 ρ^1Cancri....	6.0	K1	5.2	5.6	0.07	0.08	0.08	0.08	K.
α Boötis.....	0.2	G9	0.1	0.7	0.10	0.13	0.11	0.08	K.
W.B. 15^h720...	6.8	K3	5.9	5.0	0.07	0.04	0.06	0.05	R.
Lal. 30024.....	7.0	K3	7.5	7.3	0.13	0.11	0.12	0.09	Y.
Fed. 2895.....	6.3	G9	6.9	5.9	0.13	0.08	0.10	0.05	Y.
Pi 20^h23.....	7.3	K2	7.3	7.1	0.10	0.09	0.09	0.08	Y.

TABLE I (Continued)

STAR	MAG.	TYPE	M		PARALLAX		Mean	Measured parallax	Authority
			A	B	A	B			
K5-K9									
Lal. 1799.....	8.0	K5	7.5	6.4	0.08	0.05	0.06	0.06	Y.
Lal. 1964.....	8.6	K5	8.0	8.2	0.08	0.08	0.08	0.10	Y.
Pi 2^h123.....	5.9	K6	6.7	5.8	0.14	0.10	0.12	0.14	K.
Groom. 745....	8.3	K7	8.7	8.2	0.12	0.10	0.11	0.08	K.
α Tauri.....	1.1	K4	0.7	1.9	0.08	0.14	0.11	0.07	K.
Lal. 18115 Pr...	7.9	K8	9.0	8.7	0.17	0.14	0.16	0.16	K.
Lal. 18115 Fol..	7.9	K8	9.0	9.6	0.17	0.22	0.19	0.16	K.
Lal. 19022.....	8.2	K6	6.9	7.3	0.05	0.07	0.06	0.06	K.
A. Oe. 10603...	6.8	K8	7.9	8.3	0.17	0.20	0.18	0.18	K.
O ϵ 298.....	7.9	K6	6.8	6.9	0.06	0.06	0.06	0.05	R.
Brad. 2179....	6.7	K4	8.2	7.8	0.20	0.17	0.18	0.15	F.
W.B. 17^h322...	7.8	K7	8.6	8.9	0.14	0.17	0.15	0.12	K.
70 Ophiuchi Ft.	6.0	K7	7.8	7.6	0.23	0.21	0.22	0.17	K.
Lal. 40844.....	9.0	K5	8.8	7.9	0.09	0.06	0.08	0.17	Y.
61^1 Cygni.....	5.6	K8	8.5	7.2	0.38	0.21	0.30	0.31	K.
61^2 Cygni.....	6.3	K8	8.8	9.0	0.32	0.35	0.34	0.31	K.
Brad. 3077.....	5.6	K7	6.7	6.8	0.17	0.17	0.17	0.16	K.
Ma-Mb									
Groom. 34.....	8.2	Ma	10.4	10.3	0.28	0.26	0.27	0.28	K.
Lal. 21185.....	7.6	Ma	10.6	10.6	0.40	0.40	0.40	0.40	K.
Lal. 21258.....	8.9	Ma	10.5	10.3	0.21	0.19	0.20	0.20	K.
Lal. 25372.....	8.7	Ma	10.0	9.6	0.18	0.15	0.16	0.18	K.
Krüger 60.....	9.2	Ma	10.3	10.0	0.17	0.15	0.16	0.26	K.
Lal. 46650.....	8.9	Ma	10.3	10.1	0.19	0.17	0.18	0.18	K.

TABLE II

STAR	MAG.	TYPE	M		PARALLAX		Mean	Measured parallax	Authority
			A	B	A	B			
F0-F7									
α Can. Min.. ..	0.5	F3	+3.5	+3.0	+0".40	+0".32	+0".36	+0".32	K.
10 Urs. Maj....	4.1	F6	4.7	3.9	0.13	0.09	0.11	0.09	Y.
39 Leonis.....	5.8	F7	5.1	4.7	0.07	0.06	0.07	0.09	Y.
3 Cygni.....	6.2	F3	5.9	5.9	0.07	0.07	0.07	0.07	K.
Groom. 3042...	5.7	F0	5.4	4.8	0.09	0.07	0.08	0.06	Y.
Groom. 3357...	6.6	F7	5.5	5.5	0.06	0.06	0.06	0.08	J.
33 Pegasi.....	6.1	F5	4.7	4.2	0.05	0.04	0.05	0.06	vM.
34 Pegasi.....	5.8	F6	4.6	3.3	0.06	0.03	0.05	0.07	F.
F8-G7									
Brad. 3212.....	6.2	G7	6.7	5.9	0.13	0.09	0.11	0.15	Y.
Lal. 3022.....	7.8	G5	4.0	5.7	0.02	0.04	0.03	0.05	Y.
δTrianguli.....	5.1	F8	5.3	5.0	0.11	0.10	0.10	0.12	K.
Lal. 5490.....	6.7	G2	5.6	4.5	0.06	0.04	0.05	0.06	Y.
10 Tauri.....	4.4	F8	3.4	4.0	0.06	0.08	0.07	0.07	Y.
Groom. 884....	7.1	F9	5.8	6.7	0.06	0.08	0.07	0.09	K.
Lal. 13849.....	6.5	G7	3.7	3.6	0.03	0.03	0.03	0.11	Y.
Pi 6^h305.....	6.0	G0	5.4	4.8	0.08	0.06	0.07	0.07	K.
Groom. 1281...	5.6	F9	5.4	4.8	0.09	0 07	0.08	0.07	J.
Lal. 14146.....	7.3	F9	6.5	5.6	0.07	0 05	0.06	0.08	Y.
Brad. 1433.....	5.9	F7	4.5	4.2	0.05	0 05	0.05	0.10	J., M.
Groom. 1646...	6.5	G1	6.6	5.8	0.10	0.07	0.08	0.08	K.
Lal. 22585.....	6.4	G7	3.7	3.7	0.03	0.03	0.03	0.12	Y.

TABLE II (Continued)

STAR	MAG.	TYPE	M A	M B	PARALLAX A	PARALLAX B	Mean	Measured parallax	Authority
Lal. 25012.....	7.5	G7	7.7	7.6	0.11	0.10	0.10	0.05	Y.
Lal. 26289.....	6.3	G2	4.8	5.4	0.05	0.07	0.06	0.11	Y., J.
χ Herculis.....	4.6	F8	4.8	3.9	0.11	0.07	0.09	0.10	Y.
Groom. 2305...	6.8	G5	6.6	7.6	0.09	0.14	0.11	0.07	Y.
Lal. 30699.....	7.8	G6	5.4	4.6	0.03	0.02	0.03	0.05	Y.
26 Draconis....	5.3	G0	5.1	5.1	0.09	0.09	0.09	0.09	Y.
16 Cygni Pr....	6.3	G1	6.0	5.0	0.09	0.05	0.07	0.05	S. & M.
16 Cygni Fol...	6.4	G3	4.2	3.7	0.04	0.03	0.04	0.04	S. & M.
Lal. 38380.....	5.7	G7	4.5	3.3	0.06	0.03	0.05	0.06	Y., F.
15 Sagittae.....	5.9	G1	5.2	4.6	0.07	0.05	0.06	0.10	K.
Groom. 3150...	6.1	G0	6.9	6.4	0.14	0.11	0.12	0.12	Y.
ρ Cygni.....	4.2	G5	3.6	2.8	0.08	0.05	0.06	0.05	Y.
Fed. 4371.....	7.5	G1	7.5	6.8	0.10	0.07	0.08	0.06	Y.
107 Piscium....	5.3	K2	5.8	5.9	0.13	0.13	0.13	0.13	Y.
W. B. 2^h927....	8.2	G9	5.5	5.2	0.03	0.03	0.03	0.10	Y.
ϵ Eridani.....	3.8	K3	6.1	5.8	0.29	0.25	0.27	0.31	Y.
δ Eridani.....	3.7	G9	2.6	1.1	0.06	0.03	0.05	0.19	Y.
Lal. 10797.....	7.3	K0	5.5	5.0	0.04	0.03	0.04	0.08	Y.
Lal. 15547.....	8.6	K1	5.5	4.7	0.02	0.02	0.02	0.09	Y.
G8-K4									
83 Leonis B r...	6.5	G8	+5.8	+4.3	+0".07	+0".04	+0".06	+0".06	R.
Lal. 26196.....	7.6	K2	7.6	6.2	0.10	0.05	0.08	0.14	Y.
Lal. 27298.....	7.9	G9	6.6	7.4	0.05	0.08	0.06	0.06	K.
Lal. 27744.....	6.7	G8	7.2	5.6	0.13	0.06	0.10	0.09	F., Lee
Lal. 33439.....	6.7	K3	6.7	6.4	0.10	0.09	0.09	0.05	Y.
Lal. 38383.....	7.2	K1	7.3	7.8	0.10	0.13	0.12	0.11	Y.
η Cephei.....	3.6	K1	1.8	2.0	0.04	0.05	0.05	0.08	Y., S.
A. Oe. 25685...	7.5	K0	6.5	5.4	0.06	0.04	0.05	0.08	Y.
γ Cephei.....	3.4	K2	3.5	2.2	0.10	0.06	0.08	0.07	S. & M.
K5-K9									
Lal. 47231.....	8.6	K6	8.8	9.3	0.11	0.14	0.12	0.12	Y., S.
Lal. 1198.....	8.1	K7	6.3	5.4	0.04	0.03	0.04	0.08	Y.
Lal. 6320.....	7.9	K8	8.0	7.4	0.10	0.08	0.09	0.07	Y.
W.B. 4^h1189...	6.5	K7	7.1	6.4	0.13	0.10	0.12	0.11	Y.
Lal. 10299.....	8.5	K7	8.2	8.8	0.09	0.11	0.10	0.10	Y.
Lal. 13284.....	6.9	K5	6.6	8.3	0.09	0.19	0.14	0.12	Y.
83 Leonis Ft.. .	7.6	K7	6.3	6.5	0.05	0.06	0.06	0.06	R.
Pi 14^h212 Br...	5.8	K6	7.6	6.6	0.23	0.15	0.19	0.17	K.
Lal. 31055.....	7.9	K6	7.8	7.3	0.10	0.08	0.09	0.07	Y., F.
Lal. 34986.....	8.1	K6	7.7	7.3	0.08	0.07	0.08	0.08	Y.
B. D. $+7°3967$.	9.3	K5	8.7	7.6	0.08	0.05	0.06	0.04	Sch.
Lal. 39866.....	8.4	K3	8.4	6.8	0.10	0.05	0.08	0.06	Y.
Lal. 45028.....	7.8	K5	6.3	5.7	0.05	0.04	0.05	0.05	Y.
Ma									
Fed. 1384^1.....	9.2	Ma	9.9	10.4	0.14	0.17	0.15	0.10	F.
W. B. 16^h906..	8.8	Md	10.1	10.0	0.18	0.17	0.18	0.22	Y.
A. Oe. 17415...	9.1	Mb	10.6	10.8	0.20	0.22	0.21	0.27	K.
Pos. Med. 2164^1	8.9	Mb	10.5	10.6	0.21	0.22	0.22	0.29	K.

The columns in the tables designated by A and B refer to the determinations by Adams and Miss Burwell. The final values are the means for the two observers. The measured parallaxes are taken from a variety of sources. Y. indicates Yale determinations; K., the values compiled by Kapteyn in *Groningen Publication* No. 24; Sch., the results of Schlesinger; R., those of Russell; vM., of van Maanen; S., of Slocum; M., of Mitchell; J., of Jost; and F., those of Flint. Where relative parallaxes are given the values have been reduced to absolute measure by making suitable corrections for the parallaxes of the comparison stars. The tables are arranged according to spectral type.

The comparison of the computed and the measured parallaxes shows an excellent degree of accordance for most of the stars. There are, however, occasional large discrepancies. Of these the most serious is in the case of δ *Eridani*. The spectrum observations give a much smaller parallax than is found by the Yale observers. A striking case of agreement, on the other hand, is that of ϵ *Eridani;* this parallax was computed before it was known that a measured value was available. A star which should prove of exceptional interest is Boss 6129. From spectrum observations we have obtained a parallax of $+0''.23$: no measured value has been published but the star is on the observing programme at several observatories.

The average deviation, taken without regard to sign, between the observed and the computed values of the parallaxes in Tables I and II is $0''.024$: it is $0''.026$ for the stars of Table II alone. There seems to be no marked systematic difference between the observed and the computed parallaxes; the former average somewhat larger, but this is due mainly to a few large discrepancies.

There are 25 stars with measured negative parallaxes for which we have made spectrum determinations. The largest value for any one of these stars as computed from the line intensities is $+0''.08$; the average value for all is $+0''.03$. The spectrum method, of course, gives no negative parallaxes.

It seems reasonable to conclude from these results that the method of computing absolute magnitudes and parallaxes from the variation of the intensities of lines in stellar spectra is capable of yielding results of a very considerable degree of accuracy. Especially in the K and M type stars of low luminosity, the line variations are so great that such stars may be recognized from a mere inspection of the spectrum. Stars, for example, like 61 Cygni, Groom. 34, and Krüger 60 bear very evident marks of their intrinsic faintness in the remarkable intensity of the low temperature calcium lines in their spectra. At first thought it might

appear that a relationship between certain spectral characteristics and the distances of stars could hardly be credible, since it would appear like a correlation between two utterly unrelated subjects except in so far as the scattering of light in space might connect them. In fact, of course, it is not the distances but the absolute magnitudes of stars which have an influence on the character of the spectrum lines and such an effect, far from being improbable, is rather to be expected than not. The derivation of the distances is merely a by-product resulting from the combination of real, or absolute, with apparent magnitudes.

An important gain in the value of this method of determining stellar magnitudes and distances should result from an increase in the number of measured parallaxes of bright stars of small proper motion. Such stars will on the average prove to be very luminous, and, as already stated, the portion of the curves connecting line intensity with absolute magnitude is subject to much more uncertainty in the case of the high luminosity stars than in any of the others. It is probable that after such a revision has been made the method will find its most important application as a means of distinguishing these giant stars in the stellar system.

INVESTIGATIONS IN STELLAR SPECTROSCOPY. IV. SPECTROSCOPIC EVIDENCE FOR THE EXISTENCE OF TWO CLASSES OF M TYPE STARS

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The principal distinguishing feature of the M type of stellar spectrum on the Harvard system of classification is the presence of absorption bands due to titanium oxide. These bands increase in intensity for the successive subdivisions Ma, Mb, and Mc. The star α Orionis, in which they are present in moderate intensity, is selected as a typical Ma star by the Harvard observers. Since these bands may be seen faintly in stars of the K5 type of spectrum it is necessarily largely a matter of judgment whether in any given spectrum they are sufficiently strong to warrant classifying the star as Ma, or whether it should still be retained within the K type.

For types of spectra previous to M the principal basis of classification is the intensity of the hydrogen lines. These reach a maximum in the A type, and grow fainter in the successive types F, G, and K. Of the hydrogen lines in α Orionis, however, Miss Cannon, in the course of

her classification of the Harvard spectra, makes the statement that they are of about the same intensity as in α Tauri, a typical K5 star.[1]

The classification of the Mount Wilson stellar spectra in accordance with the Harvard system, a description of which is given in a previous communication,[2] is based upon a comparison of the intensities of the hydrogen lines with those of neighboring iron lines which are subject to relatively slight variation with type. A series of curves have been constructed giving the relationship between the relative intensities of these pairs of lines and the spectral type; and the determination of type is thus reduced to an estimation of the intensities of these lines. The stars used in the derivation of these curves are almost wholly stars of large proper motion, and in many cases have measured parallaxes of considerable size. They are, accordingly, stars of relatively low intrinsic brightness in general. This is true especially of the K5–K9 and Ma stars, nearly all of which, like 61 Cygni and Groom. 34, are of very low absolute luminosity. The curves derived in this way show a regular decrease in the intensity of the hydrogen lines throughout the range of spectrum employed, the lines in K5 stars being fainter than in K0, and in the Ma stars fainter than in K5. In fact the hydrogen lines are barely visible in most of the M stars used in the construction of the curves.

When these results are applied to the M stars of high luminosity a very anomalous condition is found. The presence of the bands places these stars definitely in the M type, but the hydrogen lines are of quite abnormal intensity. Thus α Orionis, with bands of type Ma, if classified on the basis of its hydrogen lines would become G2. This is the most remarkable case found as yet, but all of the high luminosity M stars show a strong tendency in the same direction. The results of a classification of 48 stars of types Ma to Mc on the basis of the intensities of their hydrogen lines may be summarized as follows:

TABLE I

TYPE	NO. OF STARS	TYPE	NO. OF STARS	TYPE	NO. OF STARS
G2	1	G7	9	Ma	20
G3	1	G8	10	Mb	18
G4	3	G9	4	Mc	10
G5	3	K0	4		
G6	11	K1	2		

Accordingly, the most advanced type found for any of these stars from a determination of the intensities of their hydrogen lines is K1, and the average type is G7. This is as against an average type of Mb given by

the intensities of the bands. Two conclusions may be drawn at once from these results: First, that the Harvard system of classification, in which the M type stars are all included in one group on the basis of the presence of the bands, fails entirely to discriminate between the spectral peculiarities of the high and the low luminosity M stars; and second, that the intensity of the hydrogen lines in the M stars probably varies with the absolute magnitude, the brighter stars having the stronger hydrogen lines.

A method of determining the absolute magnitudes of stars from the characteristics of certain of their spectral lines has been described in a previous communication.[3] The essential feature of this method is the use of the two lines λ 4216 of strontium and λ 4455 of calcium, the intensities of which appear to be connected directly with the intrinsic brightness of the stars in whose spectra they occur. The intensities of these lines relative to other lines in the spectrum are estimated, and a numerical relationship is established between these intensity ratios and absolute magnitude by means of a selection of stars of known parallax. In this way the following formulae applicable to stars of types G8–K4 have been derived. M is the absolute magnitude, and Δ the intensity ratio for each pair of lines.

$\frac{4216}{4250}$	$\frac{4455}{4462}$	$\frac{4455}{4494}$
$M = -1.6\Delta + 4.7$	$M = +1.6\Delta + 5.1$	$M = +2.3\Delta - 0.3$

It is this set of formulae which has been used in the case of the M stars of high luminosity. The average type of these stars was found to be G7, which is sufficiently near the limits of the group to admit of the application of the corresponding equations. Summarized briefly the results for the high and the low luminosity stars are as follows:

	Average Spectrum	*No. of Stars*	*Average M*	*Range of M*
High luminosity	G7	48	+1.4	−1.0 to + 3.4
Low luminosity	Ma	10	+10.3	+9.8 to +10.7

Of the high luminosity stars only two, α Orionis and Boss 660, have negative values of the absolute magnitude, and only five stars have values exceeding 2.0. The remaining 41 stars have magnitudes ranging between 0.0 and 2.0. It is clear, accordingly, that on the basis of absolute magnitude determinations the M stars fall into two clearly defined groups, separated by an interval of about 7 magnitudes within which no intermediate values have been found.

The spectroscopic evidence, therefore, confirms the hypothesis of Hertzsprung and Russell that the M type stars are divided into two

groups of 'giant' and 'dwarf' stars.[4] This hypothesis was based primarily on parallax observations. The absolute magnitudes calculated from these parallaxes showed almost a complete absence of stars of brightness intermediate between exceedingly luminous stars like α Orionis, and extremely faint stars such as Groom. 34. It has been thought probable by some astronomers that this apparent gap is due to the fact that parallax determinations have hitherto been restricted almost entirely to a few stars of great apparent brightness, and to stars of very large proper motion, while the connecting links would probably be found among stars of moderate apparent brightness and moderate proper motion. The spectroscopic evidence, however, is based upon numerous stars of just this character, and the gap still appears to persist.

These results may be summarized briefly as follows. Two groups of M stars are indicated clearly by an examination of the intensities of the hydrogen lines: in the first the hydrogen lines are very strong; in the second they are very faint. A computation of the absolute magnitudes of these stars on the basis of certain peculiarities in their spectra shows the existence of these groups distinctly. Connecting links over a range of 7 magnitudes are entirely lacking, and the conclusion seems to be unavoidable that among these stars the intensity of the hydrogen lines varies with the absolute magnitude.

The results given for the high and the low luminosity stars may be used to furnish an approximate relationship between the intensities of the hydrogen lines and absolute magnitude. Thus we have for $H\beta$:

	Average M	*Intensity of Hβ*
High luminosity stars	+ 1.4	+1.9
Low luminosity stars	+10.3	−3.0

Assuming a linear relationship between intensity and absolute magnitude we obtain the equation

$$M = -1.8\,\Delta + 4.8$$

This is remarkably similar to the corresponding equation found for the line λ 4216 and given on a preceding page. It seems probable, therefore, that in the case of the M stars, at least, the hydrogen lines may be used for absolute magnitude determinations in the same way as λ 4216.

There is, however, one characteristic of the spectra of these high luminosity stars which must be taken into consideration when use is made of the relative intensities of the hydrogen lines. This is a relationship which appears to exist between the intensities of the hydrogen lines and the intensities of the bands, the hydrogen lines being stronger when the

bands are stronger. There are occasional exceptions to this rule, as in the case of α Orionis, but in general the effect is well marked. Thus if we compare the intensity of the hydrogen line $H\beta$ in the stars having bands of moderate intensity with that in stars in which the bands are very strong we find the following result:

No. of Stars	*Intensity of Bands*	*Intensity of $H\beta$*
13	Moderate	+1.2
20	Strong	+1.7

It is of interest to note in this connection that the computation of the absolute magnitude shows that the Mc stars, in which the bands are exceedingly strong, are brighter on the average than those in which the bands are less intense.

Among the high luminosity stars are some with proper motions of moderate size. The absolute magnitudes of these stars should average somewhat less than those of the very small proper motion stars which constitute the remainder of the list. An analysis of the results for the 48 stars gives the following comparison. M is the absolute magnitude and m the apparent magnitude.

No. of Stars	*Average P.M.*	*Average m*	*Average M*
15	0".155	5.06	1.54
33	0.017	5.49	1.29

After making the necessary correction for the difference in the values of the average apparent magnitude we find the large proper motion stars to be about 0.7 magnitude fainter than those of small proper motion. This furnishes a check on the accuracy of the absolute magnitude determinations.

The variations in the intensities of the hydrogen lines and of the two lines used in the computation of absolute magnitude form only a part of a more general difference in the spectral characteristics of the high and the low luminosity M stars. The results of a detailed comparison of the spectrum of α Orionis (M = -1.0) with that of Lal. 21185 (M = + 10.6) and of other intrinsically faint stars may be summarized as follows:

	α Orionis	*Lal. 21185*
Enhanced lines, especially those due to *Fe*, *Ti*, *Sr*, and *Y*....	Strong	Weak
Hydrogen lines.....	Strong	Weak
Low temperature lines of *Ca*, *Ti*, *Cr*, and *Sr*.....	Weak	Strong
λ 4227 of calcium.....	Weak	Very strong

Results of a character very similar to these were found in a comparison of the spectra of α Tauri (K5) and 61[1] Cygni (K8) two stars differing

in brightness by nearly 8 magnitudes, and also in the case of the N and the R type stars of the Harvard classification. The differences, accordingly, appear to be fundamental in nature, and associated with the intrinsic brightness of the stars of the several types. They indicate a lower temperature in the absorbing gases constituting the atmospheres of the fainter stars, and are analogous in many respects to those observed in the spectrum of sun-spots.

The division of the M type stars into two well-defined classes of high and low luminosity stars raises the question at once whether a corresponding separation may be found among other types of spectra. From his discussion of parallax observations Russell concludes that such a

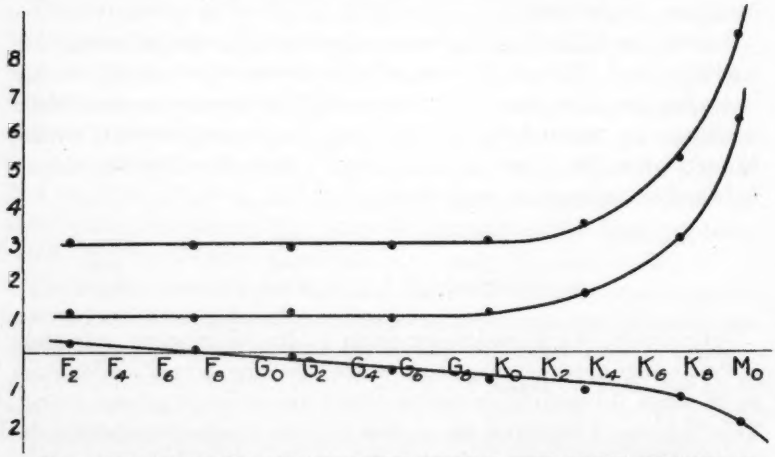

FIG. 1. CURVES SHOWING VARIATION WITH SPECTRAL TYPE OF THE RELATIVE INTENSITIES OF PAIRS OF LINES USED IN ABSOLUTE MAGNITUDE DETERMINATIONS.

Top curve, $\frac{4455}{4494}$; Middle curve, $\frac{4455}{4462}$; Bottom curve, $\frac{4216}{4250}$.

separation does exist among the K stars. The spectroscopic evidence tends to support the existence of such a division at least for the K5–K9 stars. This evidence is of just the same character as that in the case of the M type stars, and is of two kinds. First, the hydrogen lines have an abnormally high intensity in the very luminous stars, and there is an absence of intermediate values of the intensity between these and the low values characteristic of the fainter K5–K9 stars. Second, computations of absolute magnitude indicate the existence of two mean magnitudes, one high and the other low, about which the values for the individual stars showed a marked tendency to gather. This effect is not so well defined as for the M stars, but still very clear. It may perhaps

be shown to the best advantage by a reproduction of the curves representing the estimated intensity differences for the pairs of lines used in the determinations of absolute magnitude. These are given in figure 1. The curves are based upon essentially all of the stars with observed parallaxes for which we have spectral observations. Each point on the curves represents the mean for a considerable number of stars; and, as these stars differ in absolute magnitude, the corresponding intensity differences for the pairs of lines will differ. In types F and G the higher and lower luminosity values and the line differences balance one another so nearly that the successive values show but a gradual change, and the curves make but a slight angle with the horizontal axis. At about K3, however, the curves begin to bend abruptly, and the remaining types depart from the axis very rapidly. This is due to the absence of stars of even moderately high luminosity among those upon which the curves are based.

The corresponding curves for the high luminosity stars of these types run nearly parallel to the horizontal axis. We find, accordingly, both for types K5–K9 and M, a branching of the curves which points directly toward the existence of a division into two distinct groups. This evidence is based upon all of the spectroscopic material available.

In conclusion reference should be made to the necessity of adding to the symbols used in the Harvard system of classification for the M stars some character or figure which shall serve to distinguish between the spectral characteristics of the high and the low luminosity stars. The most important of these is the difference in the intensity of the hydrogen lines. Accordingly, though somewhat cumbersome in practice, I can think at present of no method which would convey the necessary information in any better way than by adding to the classification based on band intensity the corresponding classification based on hydrogen line intensity. Thus Mb (G6) would indicate a spectrum in which the bands are strong but the hydrogen lines give a type of G6. On this basis the low luminosity M stars would be of normal type and would require no suffix.

[1] *Ann. Obs., Harv. Coll.*, 28, 160.

[2] These *Proceedings*, 1, 481 (1915).

[3] These *Proceedings*, 2, 143 (1916).

[4] Summarized by Eddington in *Stellar Movements and the Structure of the Universe*, pp. 170–177.

THE FAILURE AND REVIVAL OF THE PROCESS OF PIGMENTATION IN THE HUMAN SKIN

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In the year 1914 I published an illustrated study entitled 'A Piebald Family of White Americans.'[1]

Further research shows that my conclusion then [one of 'the apparent positive results of this [that] research'], namely, that the spotting of the skin was due to 'progressive albinism,' was correct. To get the matter fairly before us I quote herewith my former conclusions, together with explanations which accompanied them. Attention is especially directed to number 4 below:

Without going further into details at this time, I may summarize the apparent positive results of this research so far in hand, as follows:

That in the family before us we see—

1. Hereditary spotting of the skin.
2. The character of spotting behaves as a simple Mendelian dominant.
3. The piebald persons are heterozygous for this character of spotting.
4. The condition of spotting is albinistic, and is progressive rather than fixed, giving progressive albinism—sometimes called dynamic leucosis.

It may be well to present here definitions of albinism in its three commonly recognized phases:

Complete albinism affords no visible pigment anywhere in *skin*, *hair* or *eyes*.

Incomplete albinism affords visible pigment of various degrees of diffusion everywhere in *skin*, *hair*, and *eyes*.

Partial or imperfect albinism affords visible pigmentation limited to areas separated by unpigmented areas. This gives 'piebald' and 'spotted' cases.

Concerning the probable close interrelation between these various phases of unpigmented skin, Pearson says:

> When we consider the relative rareness of complete albinism, of the spotted or splashed condition and of xanthism, their relatively frequent coincidence in[2] the same stock suggests that these abnormal pigment conditions are not wholly independent, and that as a working hypothesis it is reasonable to suppose that complete albinism, partial albinism, incomplete albinism, and xanthism, all static forms of leucosis, are phases of the same process and are probably linked with leucoderma and possibly other forms of dynamic leucosis. By 'linked' we suggest that they mark the complete, incomplete, local, or progressive failure of the same metabolic process, which may never start at all, never start in certain areas, or be imperfectly started, and again being started may fail to maintain itself; further, that every variety of this failure may individually or collectively be associated with certain stocks, which may either show hereditary failure of one phase, of several, or exceptionally of all phases of pigment metabolism."[3]

Pigmentation is due to *pigment metabolism*. In 'complete albinism' pigment metabolism completely fails to start. In 'incomplete albinism' pigment metabolism occurs only incompletely. In 'partial or imperfect albinism' pigment metabolism locally fails or never starts. In 'progressive albinism,' or dynamic leucosis, pigment metabolism, though having apparently once started at some time, fails in certain areas.[3]

Before giving proofs of progressive albinism as shown by the recent research, let me add another quotation from the original study which states two of the five problems then left open for further investigation:

2. Whether the albinistic areas extend their borders after once having been known, or whether there is, instead, a progressive failure of pigment metabolism within a definite area.[4]
3. Whether an at-one-time albinistic area ever revives within itself the process of pigment metabolism.[5]

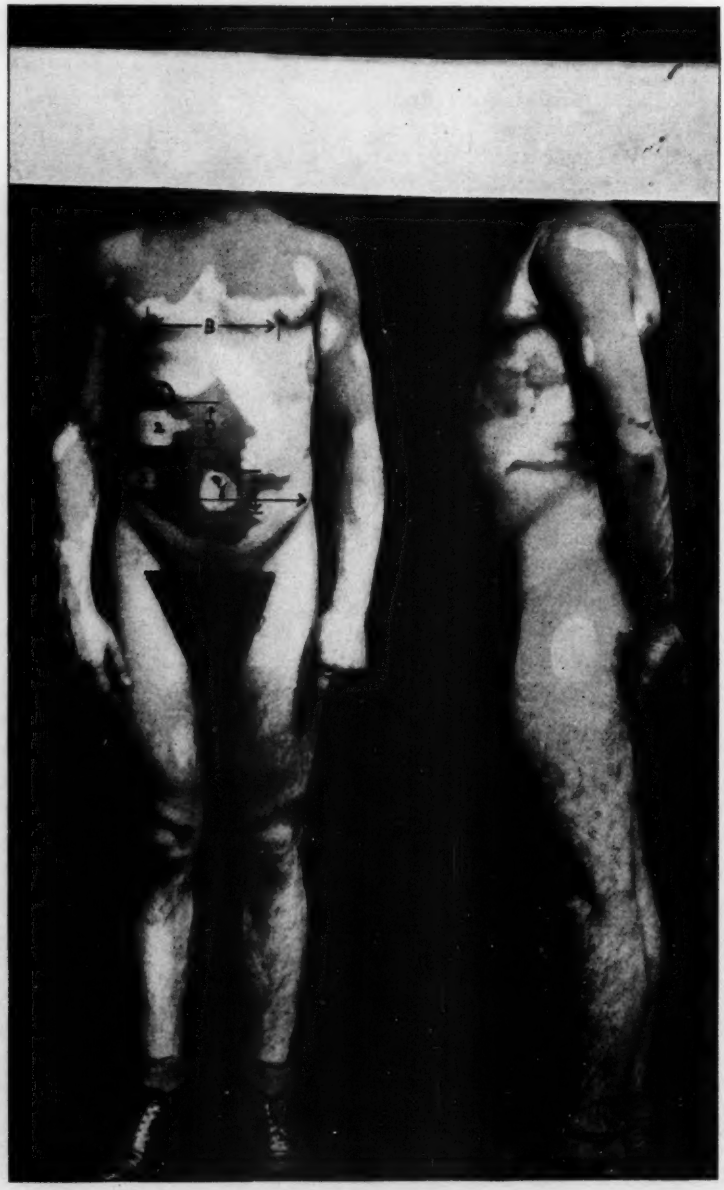
Problem number 2 has been solved. There is an extension of the albinistic areas.

Problem number 3 has also been solved, and its solution is surprising. There is a revival of the process of pigment metabolism within an at-one-time albinistic area.

The process followed in the study of the two problems (numbers 2 and 3 above) follows.

Two negatives of the same individual[6] were made as early as possible from the same points of view. The second negative was taken following a lapse of six months after the making of the first negative. The two negatives were measured by Mr. H. B. Wilcox,[7] Scholar in Astronomy at the University of Minnesota. The measurements were made on the Repsold photographic-plate measuring machine owned by the Department of Astronomy, University of Minnesota. Two lenses (one of 30 diameters, and one of 10 diameters) were used in the measurements of the distance "C" shown on Plate I. It is the horizontal measurement across the albinistic spot enclosing the navel. All subsequent measurements were made by use of the lower, or 10-diameter lens, because it resulted in better definition. However, the accuracy of the measurements by the 10-diameter lens is well confirmed by the one duplicate measure made by the 30-diameter lens, since the results obtained by the two lenses agree within the limits of error.

The error indicated is based on the theory of Least Squares—the theory of error universally employed in measurements of stellar photographs. However, attention should be called to the possibility of systematic error. This possibility of systematic error is, nevertheless, not



ABNORMALLY PIGMENTED WHITE AMERICAN SHOWING PROGRESSIVE ALBINISM

(Diagrams in black show areas measured. The black circle surrounding an "X," above area 2, has nothing to do with the present presentation.)

great, as is shown by the checking measurements made on both negatives between the points indicated by lines "A" and "B." The ratio of negative number 2 (the one made six months later than the other) to negative number 1, as shown by measurement "A" on both negatives is 1.338 ± 0.002. The ratio of negative 2 to negative 1, as shown by measurement "B" on both negatives, is 1.339 ± 0.002.

The following results were obtained by measurements of areas numbered 1, 2, and 3 on Plate I.

Results of Measurement of Albinistic Areas[8]

Area	*Measurement*	*Per cent of increase*	*Per cent of decrease*	*Per cent of error*
1	C		6.5	± .9
2	D	3.5		± .5
3	E	4.0		±1.0
1	F	1.0		±1.0

These results proclaim that the albinistic areas extended their borders, as shown by measurements "D" on area number 2, "E" on area number 3, and "F" on area number 1. The results also proclaim that the process of pigment metabolism was revived in area number 1, as shown by measurement "C," where the decrease of the size of the area was 6.5 per cent of its total height (vertical measurement of the body) with a possible error of ±0.9%.

[1] *Amer. Anthrop., New York*, N. S., 16, 222–237 (1914).
[2] *Ibid.*, p. 234. [3] *Ibid.*, p. 235.
[4] *Ibid.*, p. 233. [5] *Ibid.*, p. 234.
[6] Number 32 of the Genealogical Chart, Fig. 74, p. 222 of *Op. Cit.*
[7] Mr. Wilcox is recognized as a most careful expert in the field of stellar photographic measurements.
[8] A detailed histological study of the skin of this person may be found in the article referred to in footnote number 1.

BANDED GLACIAL SLATES OF PERMO-CARBONIFEROUS AGE, SHOWING POSSIBLE SEASONAL VARIATIONS IN DEPOSITION

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Near Boston, on the peninsula of Squantum, facing Quincy Bay, is a formation of tillite of Permo-Carboniferous age, described in 1914.[1] Lying conformably on the tillite is a slate formation with a known thickness of about 800 feet.

The manner of transition from the tillite to the slate, together with other evidence, makes it certain that the slate was the result of deposition in waters from the melting of the glacier which formed the tillite. The first of the transition beds are conglomerates alternating with sandstones. Then come sandstones alternating with slates. At several places it is plain that the ice readvanced over these beds, ploughing them up and often dragging upward into the mass thus formed irregular lumps of the clay. As one goes upward in the banded slate it is seen that the layers of sandstone become thinner and thinner, then disappear entirely, and finally the alternating layers are shown only in slate of dark and light bands. The microscope reveals the fact that the dark layers are composed of much finer material than the light layers. The coarse layers all, without exception so far as observed, have very fine wavy lines of bedding which are cut off and uneven in places, while the fine layers are solid in appearance without these characteristic lines. The finest part of the fine layer is usually in contact with the coarse layer upward, and the change from fine to coarse is abrupt. The change from coarse to fine is more gradual upward, as a rule, and not abrupt. These layers or bands alternate with much regularity and at any given horizon their thicknesses also show regularity.[2] The microscope also shows that the slate with the finest banding also has the finest particles of sediment, although the mineralogical composition is the same as in the rest of the slate and in the tillite. This finest slate however, has more chlorite than the rest of the slate, giving it a light green color.

That the slate was derived from the tillite there can be no doubt. A microscopical examination by Prof. J. E. Wolff has shown the same mineralogical composition for both. The principal minerals present are as follows: quartz, feldspar, sericite, epidote, melaphyre, chlorite, limonite, quartzite and calcite. The size of the grains in the slate range from about 1/300 of an inch to as fine as about one 1/6000 of an inch. The shapes of the grains are angular as in ordinary glacial clay particles. As already stated the light bands usually have coarser as well as lighter material, but next to the thinnest sandstone layers coarser dark material is usually found, and this is just as it should be according to the laws of deposition. Thus it is seen that the banding in this slate is determined by the specific gravity of the material as well as by the strength of current.

Identical mineralogical composition alone, however, would not prove that the slate was derived from the tillite, but two beds of tillite in the slate itself, together with the facts of the banding, prove this origin. The first bed of tillite is about 50 feet above the main tillite formation

in well banded slate. The second tillite bed is about 150 feet from the main tillite. Each of these beds is about 5 feet thick. From the evidence presented by these tillite beds and several other beds of conglomerate in the slate, it is inferred that the final retreat of the glacier was slow and hesitating, marked by several advances after many years of retreat. It is known that the ice of the Wisconsin epoch of the Pleistocene period also began its retreat very slowly.

The cause of the disappearance of the sandstone layers and the gradual thinning of the bands can be explained by the retreat of the glacier and the consequent slackening of the currents, which would be strong enough to carry sand only in the neighborhood of the ice. The lower pebbly members of the transition beds show irregularities of deposition, especially in lenticular forms, due to the inconstant conditions of streams coming from a glacier. A regularity of alternation in deposition, however, becomes evident after the first 50 feet or so of the transition beds have been passed, where the layers indicate deeper and quieter water, and thus more uniformity in deposition. The thin individual layers now show through hundreds of feet such regularity in thickness and interval that a regularly recurring cause must be sought.

At the International Congress of Geologists at Stockholm in 1910, Gerard De Geer read a paper on the banding of the glacial clays in Sweden. He thinks, that the coarse band or layer gives a record of summer melting and deposition, and that the find band or layer gives the record of winter deposition of fine material, when the streams were slow and the fine material could easily settle on the bottom. With the co-operation of his students he was able to count the layers in the late glacial clays and then in the post-glacial deposits of the extinct Lake of Ragunda, which was drained in 1796, and gives 12,000 years as the time elapsed since the ice retreated from southern Scania to its present northerly position. The idea of measuring geological time in this manner came to De Geer in 1878. In this country B. K. Emerson advanced the same idea regarding the layers in the glacial clays of the Connecticut Valley in 1898.[3] Since that time several American geologists have published the theory. Among them may be mentioned A. P. Coleman, Frank B. Taylor, and Charles P. Berkey. As far as I can learn, however, De Geer is the first to conceive this geological time recorder, and also the first to convince the larger part of the geological world that these double bands in the glacial clays really mean years. No other explanation so far advanced, accounts for all the facts of the case. The facts observed in the slates at Squantum resemble so closely those described by De Geer and Emerson and the others, that it would seem as

if there must be a very strong probability of these being similar cases, in spite of the millions of years which separate the two glacial periods.

In speaking of the banding it is not possible at this writing to say anything about the formation of slate under the main tillite. The banding in this slate appears to be somewhat different from the banding in the slate under discussion, but there has not yet been time to study it. It should show points of interest in regard to the climate responsible for the glacial advance, while the slate above the tillite should show the conditions responsible for the retreat.

Evidence of seasons during Permian times has come from New South Wales, Australia, in the discovery of well marked annual rings of growth in trees of Permian Age.[4] From Brazil similar evidence was obtained in tree trunks of the same age, and perhaps in the Triassic period which followed.[5] Such findings, together with what now appears to be good evidence of seasonal changes at the locality which is now known as Boston, would make it appear that all these localities were in a temperate zone during Permian times as now. Furthermore, the close similarity between the layers in the Pleistocene glacial clays and the layers in the slate at Squantum would make it appear that the temperatures at the close of the Pleistocene and Permian were very similar.

Not all slates which exhibit banding are of glacial origin, but a close study of the slate at Squantum should give certain definite criteria for the determination of glacial slates at other localities, and thus reveal facts hitherto unknown concerning the past climate of the earth. As glacial till is usually a land deposit and subject to rapid erosion, it is not likely to be preserved as tillite unless submerged beneath a body of water relatively soon after deposition. This is not true of a glacial slate, for slate originates in water and is thus very much more likely to be preserved than tillite. Hence, it follows that the discoveries of glacial slates should be more numerous than discoveries of tillites, and such discoveries may turn out to be as good evidences of glaciation as the tillites themselves. In this new way it may be possible to obtain very much more knowledge of past geological climates than we could ever obtain by the search for tillites alone.

[1] R. W. Sayles, The Squantum tillite, *Bull. Mus. Comp. Zool., Harvard Univ.*, **46**, No. 2, 141–175 (1914).

[3] Charles P. Berkey, Laminated Interglacial Clays of Grantsburg, Wisconsin, *The Journal of Geology*, **13**, No. 1, 36–37. January-February, 1905.

[3] B. K. Emerson, The Geology of New Hampshire County, *U. S. Geol. Surv. Monogr.*, **29**, 706 (1898).

[4] Shirley and Arber, *Queensland, New South Wales, Bull. Geol. Surv.*, No. 7, 14 (1898).

[5] I. C. White, Relatorio final apresentado A S. Ex. o Sr. Dr. Lauro Severiano Müller, Comissão de estudias das minas de Carvão de Pedra de Brazil, Rio de Janeiro, 1908. (Printed in Portuguese and English.)

AN EXTENSION OF FEUERBACH'S THEOREM

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Feuerbach's theorem, that the four circles which touch three lines also touch a circle, may be stated thus: given four orthocentric points, forming four triangles, the 16 in-circles of these triangles touch the circle F on the diagonal points.

Now each in-circle and the omitted one of the four points is a degenerate curve of class three on the absolute points $I\,J$. There is further a rational curve of class three, on the six joins of the four points and touching the line infinity at $I\,J$, which touches F three times. Thus the theorem is suggested: *All circular line-cubics on the joins of four orthocentric points touch the Feuerbach circle.*

A proof is as follows. It is convenient to state the algebra dually. That is, we have 4 lines 1, $\pm$ 1, $\pm$ 1 and a pair of lines, ζ and $1/\zeta$, apolar to all conics on the 4 lines. Two point-cubics on the six joins of the 4 lines meet again at 3 points $x\,y\,z$, which are points of contact of a tritangent conic of either cubic. When x and y are given, z is rationally known; and when x is given and z moves on a line ζ, we know from the theory of the Geiser transformation that y moves on a rational quartic ρ_2^4 which has a triple point at x. There is then a connex of the form

$$\zeta\, x^4\, y^4$$

where z is on ζ. And if ξ be the join of x and y this connex is of the form

$$\zeta\, \xi^3\, x\, y. \tag{1}$$

If ζ be 1, 0, 0, the quartic in y is the two lines

$$\begin{vmatrix} x_0 & x_1 & x_2 \\ y_0 & y_1 & y_2 \\ 0 & 1 & \pm 1 \end{vmatrix}$$

and the conic on x and the 4 other points, that is

$$(\xi_1^2 - \xi_2^2) \begin{vmatrix} x_0^2 - x_1^2 - x_2^2, & x_1 x_2 \\ y_0^2 - y_1^2 - y_2^2, & y_1 y_2 \end{vmatrix}.$$

Hence the connex is explicitly

$$\sum \zeta_0 (\xi_1^2 - \xi_2^2) \{ (x_0 y_1 + x_1 y_0)\, \xi_1 - (x_0 y_2 + x_2 y_0)\, \xi_2 + 2 (x_1 y_1 - x_2 y_2)\, \xi_0 \} = 0. \tag{2}$$

We now find where this curve in y meets the line $1/\zeta$. That is, we eliminate y from

$$\Sigma = 0, (\xi y) = 0, (y/\zeta) = 0.$$

We have then

$$\Sigma\zeta_0 (\xi_1^2 - \xi_2^2) \begin{vmatrix} x_1\xi_1 - x_2\xi_2, & x_0\xi_1 + 2x_1\xi_0, & -x_0\xi_2 - 2x_2\xi_0 \\ \xi_0 & \xi_1 & \xi_1 \\ 1/\zeta_0 & 1/\zeta_1 & 1/\zeta_2 \end{vmatrix} = 0,$$

or since $(x\xi) = 0$

$$\Sigma (\zeta_1/\zeta_2 - \zeta_2/\zeta_1) x_0 (\xi_0^2 - \xi_1^2) (\xi_0^2 - \xi_2^2) = 0$$

or if a be the join of ζ and $1/\zeta$,

$$\Sigma a_0x_0 (\xi_0^2 - \xi_1^2) (\xi_0^2 - \xi_2^2) = 0. \tag{3}$$

The values of ξ common to this equation and $(x\xi) = 0$ give the intersections y of ρ_x^4 and $1/\zeta$. Thus when $(x\xi)$ is a line of (3) then as z moves on ζ the curve ρ_x^4 touches $1/\zeta$ at the point y, and x is on the envelope sought.

Now the quartic (3) is two conics on the lines $1, \pm 1, \pm 1$. And when

$$\Sigma \sqrt{a_ix_i} = 0, \tag{4}$$

the two conics become one conic R whose equation is

$$\Sigma \sqrt{a_ix_i}\, \xi_i^2. \tag{5}$$

The conic (4) occurs then twice in the envelope, the other factors being

$$[x_0^4 + 2x_1^2x_2^2]^2,$$
$$x_0x_1x_2,$$

and the cubic of the system with a double point at a, namely

$$\Sigma x_0\eta_1\eta_2$$

where η is the join of x and a.

The conic (4) is the Feuerbach conic F, for it is on the diagonal lines of the four lines, and having the line equation

$$\Sigma a/\xi = 0,$$

it is on ζ and $1/\zeta$.

The construction of the cubics which touch both ζ and $1/\zeta$ is then as follows. Take a point x on F, and draw from x the two tangents to the conic R. The diagonals of this line-pair and the pair ζ and $1/\zeta$ give the points of contact of the two cubics. If the diagonals meet at b,

then x and b are apolar to ζ and $1/\zeta$; and the line bx, being the polar of a as to R, has the equation

$$\Sigma\, ay/\sqrt{ax} = 0,$$

and is the tangent of F at x.

Dually then, ζ and $1/\zeta$ being the absolute points, the conic F the Feuerbach circle, and the conic R a rectangular hyperbola on the four given orthocentric points, and having its centre c on F, if the common diameter of F and R meet R at points dd', then these points are double foci of circular curves of class 3 on the 6 lines; the circles with centres d and d' and touching F at c are the tritangent conics; and the two cubics touch F at c.

DEFORMATIONS OF TRANSFORMATIONS OF RIBAUCOUR

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When a system of spheres involves two parameters, their envelope consists in general of two sheets, say Σ and Σ_1, and the centers of the spheres lie upon a surface S. A correspondence between Σ and Σ_1 is established by making correspond the points of contact of the same sphere. In general the lines of curvature on Σ and Σ_1 do not correspond. When they do, we say that Σ_1 is in the relation of a transformation of Ribaucour with Σ, and vice-versa. For the sake of brevity we call it a transformation R.

It is a known property of envelopes of spheres that if the surface of centers S be deformed and the spheres be carried along in the deformation, the points of contact of the spheres with their envelope in the new position are the same as before deformation.[1] Ordinarily when S for a transformation R is deformed, the new surfaces Σ' and Σ'_1 are not in the relation of a transformation R. Bianchi[2] has shown that when S is applicable to a surface of revolution, it is possible to choose spheres so that for every deformation of S the two sheets of the envelopes of the spheres shall be in the relation of a transformation R, and this is the only case in which S can be deformed continuously with transformations R preserved. The only other possibility is that in which it is possible to deform the surface of centers of a transformation R in one way so that the sheets of the new envelope shall be in the relation of a transformation R. It is the purpose of this paper to determine this class of transformations R.

Let x, y, z denote the cartesian coordinates of Σ, expressed in terms of two parameters u and v, such that the curves $u = \text{const.}$, $v = \text{const.}$ are the lines of curvature on Σ and let the linear element of Σ be written $ds^2 = Edu^2 + Gdv^2$. If X, Y, Z are the direction-cosines of the normal to Σ and ρ_1, ρ_2 its principal radii of normal curvature, we have the equations of Rodriques

$$\frac{\partial x}{\partial u} + \rho_1 \frac{\partial X}{\partial u} = 0, \qquad \frac{\partial x}{\partial v} + \rho_2 \frac{\partial X}{\partial v} = 0, \tag{1}$$

and similar equations[3] in y and z.

Darboux[4] has shown that the most general transformation R of Σ is given by taking for S the surface whose coordinates x_0, y_0, z_0 are given by

$$x_0 = x - \frac{\lambda}{\mu} X, \qquad y_0 = y - \frac{\lambda}{\mu} Y, \qquad z_0 = z - \frac{\lambda}{\mu} Z,$$

where λ and μ satisfy the equations

$$\frac{\partial \lambda}{\partial u} + \rho_1 \frac{\partial \mu}{\partial u} = 0, \qquad \frac{\partial \lambda}{\partial v} + \rho_2 \frac{\partial \mu}{\partial v} = 0, \tag{2}$$

and by taking λ/μ for the radius of the corresponding sphere.

If x_1, y_1, z_1, denote the coordinates of Σ_1, and X_1, Y_1, Z_1; X_2, Y_2, Z_2, the direction-cosines of the tangents to the curves $v = \text{const.}$, $u = \text{const.}$ respectively on Σ, we have relations of the form

$$x_1 = x - \frac{1}{\sigma m} (\alpha X_1 + \beta X_2 + \mu X),$$

where m is a constant, and α, β, σ are functions which are in the quadratic relation

$$\alpha^2 + \beta^2 + \mu^2 = 2m\lambda\sigma,$$

and satisfy the equations

$$\frac{\partial \lambda}{\partial u} = \sqrt{E}\,\alpha, \qquad \frac{\partial \lambda}{\partial v} = \sqrt{G}\,\beta,$$

$$\frac{\partial \mu}{\partial u} = -\frac{\sqrt{E}}{\rho_1}\alpha, \qquad \frac{\partial \mu}{\partial v} = -\frac{\sqrt{G}}{\rho_2}\beta, \tag{3}$$

$$\frac{\partial \alpha}{\partial u} = -\frac{1}{\sqrt{G}} \frac{\partial \sqrt{E}}{\partial v} \beta + \mu \frac{\sqrt{E}}{\rho_1} + 2m\sqrt{\lambda\sigma}\cosh\omega.$$

$$\frac{\partial\alpha}{\partial v} = \frac{\beta}{\sqrt{E}}\frac{\partial\sqrt{G}}{\partial u}, \qquad \frac{\partial\beta}{\partial u} = \frac{\alpha}{\sqrt{G}}\frac{\partial\sqrt{E}}{\partial v},$$

$$\frac{\partial\beta}{\partial v} = -\frac{1}{\sqrt{E}}\frac{\partial\sqrt{G}}{\partial u}\alpha + \mu\frac{\sqrt{G}}{\rho_2} + 2m\sqrt{\lambda\sigma}\sinh\omega, \tag{3}$$

$$\frac{\partial\log\sigma}{\partial u} = \left(2\sqrt{\frac{\lambda}{\sigma}}\cosh\omega - \sqrt{E}\right)\frac{\alpha}{\lambda}, \quad \frac{\partial\log\sigma}{\partial v} = \left(2\sqrt{\frac{\lambda}{\sigma}}\sinh\omega - \sqrt{G}\right)\frac{\beta}{\lambda},$$

$$\frac{\partial\omega}{\partial u} = \frac{1}{\sqrt{E}}\frac{\partial\sqrt{G}}{\partial u} - \frac{\alpha}{\sqrt{\lambda\sigma}}\sinh\omega, \quad \frac{\partial\omega}{\partial v} = \frac{1}{\sqrt{G}}\frac{\partial\sqrt{E}}{\partial v} - \frac{\beta}{\sqrt{\lambda\sigma}}\cosh\omega.$$

This system of equations is completely integrable provided that

$$\frac{\partial}{\partial u}\left(\frac{1}{\sqrt{G}}\frac{\partial\sqrt{E}}{\partial v}\right) = \frac{\partial}{\partial v}\left(\frac{1}{\sqrt{E}}\frac{\partial\sqrt{G}}{\partial u}\right).$$

This condition characterizes surfaces with the same spherical representation of their lines of curvature as isothermic surfaces. Then we establish the fundamental theorem:

In order that the surface of centers of a transformation R admits a deformation into the surface of centers of a transformation R, it is necessary that the sheets of the envelope of spheres have the same spherical representation of their lines of curvature as isothermic surfaces, and every surface of this sort admits such transformations.

From (1) and (2) it follows that μ is a solution of the differential equation satisfied by X, Y, Z, and consequently when a transformation R of a surface Σ is known, a transformation R of a surface $\bar{\Sigma}$ with the same spherical representation of its lines of curvature as Σ is determined by μ, the corresponding function λ being given by equations analogous to (2) and the other functions follow from equations of the form (3). We say that the new transformation R is obtained from the given one by a transformation of Combescure. Hence if we determine the solutions of our problem for which Σ is an isothermic surface, the others may be obtained by transformations of Combescure.

When Σ is isothermic, we may take

$$\sqrt{E} = \sqrt{G} = e^{\varphi},$$

in which case the integral of the last two of equations (3) is

$$e^{\omega} = \frac{\sqrt{\lambda\sigma}\,e^{\varphi}}{\lambda + c} \tag{4}$$

where c denotes a constant of integration.

When we take $c = 0$ in (4), and substitute this value of ω in (3), the latter reduced to the equations of transformations D_m of the isothermic surface Σ into a new isothermic surface Σ_1.[5] In this case the linear element of the sheet Σ' of the new envelope arising from the deformation of the surface of centers S of the transformation D_m is

$$ds^{12} = \left(e^{\varphi} - \frac{\lambda}{\sigma} e^{-\varphi}\right)^2 du^2 + \left(e^{\varphi} + \frac{\lambda}{\sigma} e^{-\varphi}\right)^2 dv^2,$$

and the linear element of its spherical representation is

$$d\sigma^{12} = \left(\frac{e^{\varphi}}{\rho_1} + \frac{\mu}{\sigma} e^{-\varphi}\right)^2 du^2 + \left(\frac{e^{\varphi}}{\rho_2} - \frac{\mu}{\sigma} e^{-\varphi}\right)^2 dv^2. \tag{5}$$

The other sheet of the envelope, namely Σ'_1, is merely a point, say O. Conversely as we have shown elsewhere[6] also, a D_m is the only transformation R whose surface of centers can be deformed so that one of the sheets of the new envelope is a point.

From our fundamental theorem it follows that (5) is the spherical representation of the lines of curvature of two isothermic surfaces $\bar{\Sigma}$ and $\bar{\Sigma}'$, themselves Christoffel transforms of one another. Their respective linear elements are

$$d\bar{s}^2 = \sigma^2 e^{2\varphi}(du^2 + dv^2), \qquad d\bar{s}'^2 = \frac{e^{-2\varphi}}{\sigma^2}(du^2 + dv^2).$$

When we apply to $\bar{\Sigma}$ and $\bar{\Sigma}'$ transformations D_m determined by the function μ which is involved in the transformation from Σ' into the point O, we get two new isothermic surfaces $\bar{\Sigma}_1$ and $\bar{\Sigma}'_1$, whose linear elements are respectively

$$d\bar{s}_1^2 = \lambda^2 e^{-2\varphi}(du^2 + dv^2), \qquad d\bar{s}_1'^2 = \frac{e^{2\varphi}}{\lambda^2}(du^2 + dv^2).$$

The surface $\bar{\Sigma}_1'$ is the one which Bianchi[7] defined in a purely intrinsic manner and called the transform of $\bar{\Sigma}$ by the transformation T_m determined by the D_m of Σ into Σ_1.

Let S_0 be the surface of centers of the spheres enveloped by Σ' and O. Corresponding normals to Σ', $\bar{\Sigma}$, $\bar{\Sigma}'$ are parallel, and in like manner the normals to $\bar{\Sigma}_1$ and $\bar{\Sigma}'_1$ are parallel to the lines joining O to the points on S_0. Hence the permanent conjugate system on S_0, that is the system corresponding to the lines of curvature on Σ, project upon the unit sphere with center at O into the orthogonal curves representing the lines of curvature on $\bar{\Sigma}'_1$.

The surfaces $\overline{\Sigma}'$ and $\overline{\Sigma}'_1$ are in the relation of a transformation D_m. Let $\overline{S}$ be the surface of centers of the spheres of this transformation and $\overline{S}_0$ the deform of $\overline{S}$. Applying the above method to these surfaces, we find that the lines joining O to the points of $\overline{S}_0$ are parallel to the normals to Σ. Moreover, the surfaces $\overline{S}$ and S_0 correspond with parallelism of tangent planes; likewise $\overline{S}_0$ and S.

When $c \neq 0$ in (4), the surface Σ'_1 is an isothermic surface in the relation of a transformation T_m with Σ.

[1] Bianchi, *Geometria Differenziale*, 2, 88.
[2] l. c., p. 117.
[3] Eisenhart, *Differential Geometry*, 122.
[4] *Théorie Générale des Surfaces*, 2, 383.
[5] Darboux, *Ann. École Normale Supérieure*, ser. 3, 16 (1899).
[6] *Roma, Rend. Acc. Lincei*, ser. 5, 24, 161 (1915).
[7] *Ann. Mat., Milano*, ser. 3, 12, 19 (1906).

GEOGRAPHIC HISTORY OF THE SAN JUAN MOUNTAINS SINCE THE CLOSE OF THE MESOZOIC ERA

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The San Juan Mountains are located in the southwestern corner of Colorado. From their lofty summits one may look far off into Utah, Arizona, and New Mexico. The highest peak in the range, Uncompahgre, rises to an elevation of 14,300 feet above sea level. There are many other peaks whose summits are above 14,000 feet in elevation, and a large portion of the mountain area is more than 12,000 feet above sea level.

The core of the San Juan Mountains is composed of a complex series of pre-Cambrian gneisses, schists, quartzites, and granites, all of which have been more or less affected by intrusive bodies of igneous rocks. After this pre-Cambrian complex was so eroded that the relief in the region was low, the entire district was submerged and a great series of Paleozoic and Mesozoic strata was deposited over the once mountainous area.

At the close of the Mesozoic era and the opening of the Cenozoic era there were mountain-making movements which affected the entire Rocky Mountain province of North America, and the great dome which was then formed in the San Juan area was at once subjected to vigorous erosion. As the mountain mass rose erosion began, and as the great dome was more and more deeply dissected a mountain topog-

raphy must have been produced, and those mountains may be thought of as the first generation of the San Juan range (see fig. 1). During the period of mountain growth there was some volcanism. Many porphyritic intrusions and the deposition of the great volcanic tuffs which made contributions to the Ridgway till date back to this period. The Eocene till indicates that during the dissection of these early San Juan Mountains ice formed in the range and descended to the bordering lowlands. Possibly ice formed in neighboring ranges and approached the San Juan Mountains, and possibly there were distinct glacial epochs in that period of glaciation.

After the retreat and disappearance of the early Tertiary ice, stream erosion continued, and the western portion of the San Juan Mountain area was reduced to a surface of slight relief which may be thought of as a peneplain.[1] This peneplain bordered on the west a higher area of mountainous character, which supplied the material for the Telluride

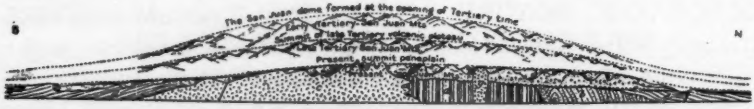

FIG. 1. DIAGRAMMATIC CROSS SECTION SHOWING THE PHYSIOGRAPHIC EVOLUTION OF THE SAN JUAN REGION SINCE THE BEGINNING OF TERTIARY TIME. *a*, PRE-CAMBRIAN SCHIST AND GNEISS; *b*, ALGONKIAN QUARTZITES; *c*, PRE-CAMBRIAN GRANITE INSTRUSION; *d*, PALEOZOIC AND MESOZOIC ROCKS; *e*, RIDGWAY TILL AND TELLURIDE CONGLOMERATE; *f*, MIDDLE AND LATE TERTIARY VOLCANICS; *g*, TERTIARY INTRUSIVES.

conglomerate. The deposition of gravels upon this peneplain surface was probably due to some uplift and rejuvenation of the streams in the eastern portion of the range. After the deposition of the Telluride conglomerate there was further erosion in the range, and then came the three great epochs of volcanism, the San Juan, the Silverton, and the Potosi. During these epochs of volcanism a great volcanic plateau was developed. By this time the Miocene epoch had been reached and possibly passed, and with the quieting down of volcanic activity began the erosion and dissection of the volcanic plateau. During this period of dissection another generation of San Juan Mountains was carved, this time out of volcanic débris and great lava flows (see fig. 1).

Recent physiographic studies of the range have led to the recognition of a summit peneplain in the San Juan region (see fig. 1), so it would appear that the first dissection stopped at a plain which is now represented by many of the summit areas within the region. The San Juan Mountains that were first carved out of this great volcanic plateau should then be thought of as surmounting those of today. Perhaps, if

replaced, they would rise 3000 or 4000 feet above the present summits (see fig. 1).

With the redoming of the area, which involved the warping or doming of the summit peneplain, another cycle of erosion was begun. Valleys were again formed, and in these valleys snows collected which in time formed glaciers that advanced to the lowlands bordering the range. These earliest Pleistocene glaciers retreated and disappeared. The range continued to be uplifted, and the streams were so rejuvenated that they cut great canyons below the broad troughs occupied by the Cerro glaciers. Again climatic changes favored the formation of ice among the summits, and that ice (the Durango glaciers) descended through the main canyons to the foothills and later retreated and disappeared. The canyons were still more deeply cut into the mountain mass, and then climatic conditions favorable for glaciation once more returned and the Wisconsin or third series of Pleistocene glaciers formed and descended through the great canyons, nearly as far as those of the Durango stage. These glaciers have now disappeared, and there is no true glacier ice remaining in the region today, but the streams are vigorously dissecting the mountain mass to still greater depths. The vigor of that work is illustrated in many a sharp, V-shaped notch cut below the depth of ice action. The débris taken from the mountin area is being distributed along the great valleys leading away from the range.

The ice gouging of the three successive Pleistocene stages and the vigorous stream work during the interglacial intervals and since the last melting away of the ice suggest somewhat continuous mountain growth in this region during late geological time.

The physiographic study of the San Juan region has associated with it much that is of human interest. As the region has passed through successive cycles of erosion the position of the ground-water table in the great mountain mass has necessarily changed. The base of the surface zone of oxidation has therefore shifted. With the deepening of erosion and the drawing off of ground waters larger portions of the ore bodies have come to be within the surface zone of oxidation. Secondary enrichment may, therefore, have proceeded to greater and greater depths as the erosion history has progressed. This suggestion has come from the study of the physiographic history of the San Juan Mountains, but as secondary enrichment seems to have been of minor importance here, the idea must be tested in other regions that have similarly passed through several cycles of erosion, and where the secondary enrichment of ore bodies has been recognized as of greater significance.

Settlement was first made in the mountains by those seeking the

mineral deposits, and the largest settlements now are mining towns. They are Ouray, Telluride, Silverton, Lake City, Creede, Rico, and Durango. About the margin of the range where agriculture has become of greater importance, and where fruit raising has been especially developed, are the towns of Montrose, Ridgway, Mancos, Ignacio, and Del Norte.

As the lowlands bordering the range have been taken up for settlement, and the broader valley bottoms within the mountains have been fenced in for ranch lands, there has come a demand for a large amount of water for irrigation purposes. Glacial lakes, in many instances, have been modified so as to form great reservoirs, and certain of the canyons have been dammed so that the surplus waters from the melting snows and heavy rains may be stored and held until late in the growing season.

The mountain slopes, where they are not too steep or too high, are clothed with magnificent forests. These are now under national control, and the ranchmen are assigned portions where they may graze their cattle, and in other portions, especially in the higher mountain areas and often in the glacial cirques or basins, thousands and thousands of sheep may be cared for during the summer. In return for this a fee is paid to the national government on a per capita basis. Where timber is cut either by the settlers or by lumbermen for commercial purposes, it is cut under the supervision of those in charge of the national forests.

At the southwest base of the range is that region of remarkable human interest now set aside as the Mesa Verde National Park. It is one of the homes of the ancient cliff dwellers. The physiographic study of the Mesa Verde region indicates that the canyons in which those people built their homes were cut during and since the last glacial period. It is evident from the study of those homes that hundreds of not thousands of people once lived on this mesa. Today there is but a single spring, and that is near one of the smaller settlements. The articles found about these homes indicate that the people raised corn and cotton, and found in the chase an abundance of deer. Today the region is without game, and all the evidence available indicates that when the region was populated there was a much more moist climate. That does not prove that the cliff dwellers occupied those homes while there were glaciers in the mountains, but it is possible that our southwest country was inhabited by men at that time. The records from anthropology and geology are coming closer and closer together, and the further study of this portion of the continent from both points of view may lead to discoveries of unusual interest.

The study of the geography of this region has necessarily a very close relationship to the geologic studies of the range, but it may lead also to a study of anthropogeography, and to a study of how the present geographic conditions are influencing the settlement and commercial development within this district.[2]

[1] Whitman Cross, U. S. Geol. Survey Geol. Atlas, Silverton folio (No. 120), 1905.

[2] This study is here published with the permission of the Director of the U. S. Geological Survey.

THE AGE OF THE MIDDLE ATLANTIC COAST UPPER CRETACEOUS DEPOSITS

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Distribution.—The Upper Cretaceous formations of the Middle Atlantic Coast area are most extensively developed in New Jersey from which state they thin rapidly to the northward through the islands off the New England coast, only a few remnants being left in southern Massachusetts; while to the south they gradually disappear, only the lowermost reaching the Potomac drainage basin. Beyond this point they have only been recognized in deep-well borings in Virginia although the deposits probably extend continuously beneath the mantle of Tertiary formations until they again appear in surface outcrops in the South Atlantic area.

Divisions.—Several formations have been recognized within the area of outcrop and have been designated under the names of Raritan, Magothy, Matawan, Monmouth, Rancocas, and Manasquan.

Numerous minor subdivisions have been described in New Jersey but have been unrecognized elsewhere. All the deposits are of shallow-water origin as shown by their contained faunas. The relatively slight differences in the faunas of these smaller units are evidently due to the varying proximity of stream mouths and sediment-bearing currents rather than to any differences in depth which may have existed.

The major divisions, on the other hand, are based on distinct differences in the faunas and floras which may be recognized when present not only in the Middle Atlantic Coast area but throughout the South Atlantic and Gulf regions as well.

The Raritan and Magothy formations are clearly separable on the basis of their contained floras while the Matawan and Monmouth, the only marine divisions with equivalents in the South Atlantic and Gulf

regions, are separable by the initiation in the Monmouth of a new fauna marked by the introduction of *Belemnitella americana*, *Exogyra costata*, *Turritella vertebroides*, *Anchura pennata*, *Eutrephoceras dekayi*, and probably of *Liopistha protexta* which have been made the basis for the differentiation of the two primary zones shown on the Federal Survey maps throughout this region. While it is true that several of the Matawan forms persist far into the Monmouth this does not in the last detract from the significance of the initiation of a new element of more than local importance at the opening of the Monmouth. The time of extinction of an old fauna is properly considered as less significant than the time of initiation of a new one, and if that new one be sufficiently virile to characterize the molluscan life from New Jersey south through Georgia and west to Texas it indicates something more than a minor oscillation in a restricted area and should be given the relatively higher rank which it deserves. The Rancocas fauna is very distinct from the preceding Monmouth but it is only known in the restricted area of New Jersey and Delaware. The same is also true in the case of the Manasquan which has an even more limited development in New Jersey. These two later formations have no equivalents in the South Atlantic and Gulf areas.

Correlation.—Raritan.—The Raritan formation of the Middle Atlantic Coast contains a flora of over 200 species, on the relations of which its correlation must be based since but a few uncharacteristic invertebrates have been found in these beds. The Raritan flora has been chiefly recorded from New Jersey where the sediments were more favorable for its preservation than farther southward. It is clearly separable into older and younger florules, the latter making its appearance in the upper beds near South Amboy, New Jersey, ranging southward to the Potomac, and reappearing in the lower part of the Tuscaloosa formation of the Eastern Gulf and the Woodbine formation of the Western Gulf. The Raritan flora is conclusively correlated with the Cenomanian stage of the European section by reason of a number of forms common to such well-known Cenomanian floras as those of Niedeschoena, Saxony; Moletein, Moravia; Perutz, Bohemia; and Alcantara, Portugal. In addition to this considerable element common to the old world Cenomanian a large number of identical genera have closely related species in the two regions and many new and identical types appear at this horizon on both sides of the Atlantic.

Magothy.—The Magothy formation of the Middle Atlantic coast has furnished a rather poor fauna, principally from the New Jersey area, and an extensive flora of upwards of 300 species found from Marthas Vineyard, Massachusetts, to the Potomac. The Magothy is separated

from the Raritan by an erosional unconformity. Its fauna has furnished but meager data for exact correlation but the large flora furnishes decisive evidence on this point. Compared with the underlying Raritan it may be noted that 61% of the Raritan flora becomes extinct before Magothy time and 70% of the flora of the latter, including many new and progressive types, is not found at horizons as old as the Raritan. Some of the most characteristic Magothy forms are found in the Mid-

Table Showing the Approximate Correlation of the Middle Atlantic Coast Upper Cretaceous Formations with those of other Areas

N. J.—MD.	N. C.—S. C.	EASTERN GULF	WESTERN GULF	WESTERN INTERIOR	EUROPE
Manasquan					
				Laramie	Danian
Rancocas					
Monmouth	Peedee	Ripley / Selma	Navarro / Taylor	Montana	Senonian
Matawan	Black Creek	Eutaw	Austin / Eagle Ford	Colorado	Turonian
Magothy	Middendorf	Tuscaloosa	Woodbine	Dakota	
Raritan			Washita		Cenomanian

dendorf and Black Creek beds of the Carolinas, and in the upper Tuscaloosa and lower Eutaw of the Eastern Gulf.

Compared with European floras that of the Magothy shows several types identical with and many closely related to those of the lower basin of the Rhone, Bussaco, in Portugal, and the richly plant-bearing Turonian section of Bohemia. This floral similarity as well as the stratigraphic position of the Magothy leads to the conclusion that the Magothy formation is of Turonian age.

Matawan.—The earliest fauna of any extent occurs in the Matawan from which 175 to 185 species have been described in New Jersey and Maryland. Five faunal zones have been differentiated in north-central New Jersey, where the Matawan is best developed, but it has not been possible to carry any but the lowest of these south of the Delaware-Maryland line. The Merchantville, together with the Woodbury which merges into it in southern New Jersey, has been recognized along the Chesapeake and Delaware Canal just east of the state line. Although this fauna is a small one of only about 50 species, it contains the elements of one of the most widely distributed and best characterized biotas of the entire Upper Cretaceous, a fauna represented in the *Mortoniceras subzone* of the South Atlantic and Gulf states, the Austin Chalk of Texas, the Niobrara of the Western Interior, and the Emscher beds of north-central Europe.

The late Matawan deposits were probably laid down in more shallow waters. Their contained faunas are consequently more restricted in their distribution and less cosmopolitan in their affinities. The Marshalltown of New Jersey which is best characterized by the abundance of the ponderous Ostreids, such as Exogyra and Gryphaea, is quite possibly represented in the Matawan oyster banks along the Chesapeake and Delaware Canal west of St. Georges. These beds are doubtless the time equivalent of at least a part of the *Exogyra ponderosa zone* of the South Atlantic and Gulf states, a faunal zone which has been recognized in the upper Black Creek of North and South Carolina, the lower Ripley of Georgia and eastern Alabama, the lower Selma of central and western Alabama, Mississippi, and Tennessee, and the upper Eutaw of Mississippi and Tennessee.

Monmouth.—The Monmouth formation has furnished the most prolific and diversified fauna of any of the Upper Cretaceous, over 250 species having been determined and of these about 75% are peculiar.

Three faunal zones have been differentiated in New Jersey but it is impossible to trace the New Jersey horizons southward into Maryland with any assurance. The fauna of the Sassafras River area, although very prolific, is very poorly preserved and the determinable species are none of them diagnostic of a particular facies. However, the abundance of *Belemnitella americana*, in the Eastern Shore deposits of Delaware and Maryland suggests the Navesink, while *Sphenodiscus* is peculiarly characteristic both of the Tinton of New Jersey and the prolific Prince George's County fauna in Maryland. It is exceedingly improbable, however, that there is any appreciable time interval involved in these faunal differences.

The general aspect of the Monmouth biota is more southern than that of the Matawan and it is probable that the slight depression of the seas which initiated the Monmouth broke down the barriers which had earlier prevented free communication with the inshore life of the south Atlantic.

The Monmouth is at least the partial time equivalent of the *Exogyra costata zone* which has been recognized in the Peedee sand of North and South Carolina and in the Ripley and Selma chalk of the Gulf states. A number of identical species occur in the Fox Hills of the Western Interior while the ensemble is very similar to that of the Upper Campanian of the Belgian border and the so-called Maestricht beds.

Rancocas.—Two horizons have been recognized in the Rancocas of New Jersey, the Hornerstown glauconitic marl and the Vincentown "yellow sand" but these divisions are absent south of the Delaware Bay. The diagnostic features of the fauna developed in Delaware are essentially those of the Vincentown—a prolific bryozoan fauna with *Terebratula harlani* in abundance and a very meager molluscan representation. Only about 15 species have been determined, all of them bivalve, over half of which are restricted to the Rancocas. Five have been found at older horizons, while *Ostrea vomer* and *Terebratula harlani* have been reported as surviving the break between the Mesozoic and Cenozoic although there is doubt in both cases as to the identity of the species.

The mollusca of Delaware are curiously dissimilar from those of New Jersey, none of the few characteristic species of New Jersey occurring in Delaware, while the abundant Delaware *Gryphaea* to which the characteristic Vincentown bryozoa attach themselves, is apparently not present in New Jersey.

There is no closely allied fauna in this country and its closest analogue is apparently in northern Europe. Although there is little direct evidence for its correlation with the Danian yet the general facies of the two faunas is very similar. Both are characterized by an extensive bryozoan fauna and the absence of *Belemnitella*. At the same time the fauna is not Tertiary in aspect and it seems improbable that it can belong to that horizon.

Manasquan.—The Manasquan is closely related to the Rancocas both faunally and stratigraphically. Its fauna is not characteristically Cretaceous although the general assemblage of forms suggests its association with that division rather than with the Tertiary. It is regarded as Danian. It has no equivalents as far as known among American formations elsewhere.

Conclusions.—The several Upper Cretaceous formations of the Middle Atlantic Coast represent all of the major divisions of the European series. The Raritan must be regarded from its contained flora as Cenomanian while the flora of the Magothy is considered Turonian. The fauna of the lower Matawan is regarded as lower Senonian or Emscherian because of the presence of *Mortoniceras* and associated forms. Equivalent strata to the Magothy and Matawan combined are found in the Carolinas in the Black Creek formation where beds containing the Magothy flora are interstratified throughout the upper part of the formation with beds containing a Matawan fauna. The evidence is apparently conflicting and the dividing line between the Turonian and Senonian is therefore placed midway in the Matawan although this must be regarded as a compromise position. The Monmouth fauna is Senonian in age and probably represents the middle and upper Senonian. The Rancocas and Manasquan are regarded as Danian.

UPPER CRETACEOUS FLORAS OF THE WORLD

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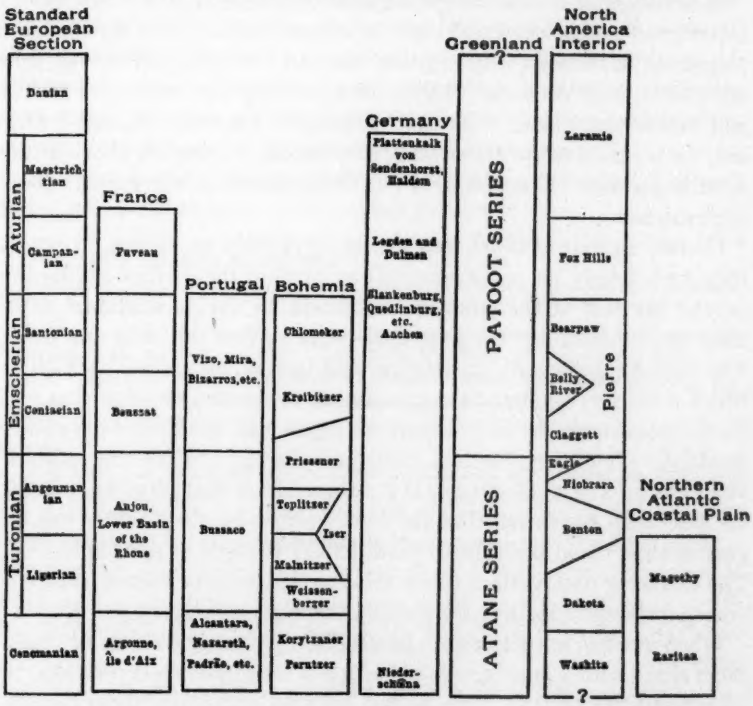
The writer has prepared a detailed account of the Upper Cretaceous floras of the world for a report on the Upper Cretaceous of Maryland which will be published later by the Geological Survey of that State.

Since the question of the age relations of the Upper Cretaceous floras of the United States has been the occasion of considerable discussion in recent years it is believed that a summary of the contemporaneous floras from other parts of the world will prove useful to paleontologists and geologists.

The stratigraphic position of the more important of these is indicated in the accompanying diagram.

The typically marine series of sediments developed in most European countries has furnished little that is of interest to the paleobotanist. The floras of the Turonian of the lower basin of the Rhone, and of the Coniacian and Campanian in the same region are well represented but largely undescribed, while the earlier Upper Cretaceous floras of France are scattered and unimportant. In Portugal extensive and important floras are intercalated in a fossiliferous marine series ranging from the Cenomanian through the Turonian and Emscherian, but these also are largely undescribed. Bohemia furnishes a splendid section from the base of the Upper Cretaceous through the Emscherian, with extensive

and fully described floras in the Cenomanian, Turonian and Emscherian that are of the utmost value for purposes of correlation. Germany furnishes an extensive flora of Cenomanian age and scattered floras ranging upward to horizons as high as the Laramie of the western United States. The upper Cretaceous floras of the west coast of Greenland are very extensive but the deposits are largely continental in type and the



floras offer difficulties to precise correlation in the upper part of the section since collections were not made from carefully ascertained levels.

There are also some representation of the floras of the Upper Cretaceous in Japan and in the southern hemisphere (Patagonia, Graham Land, Australasia) but these are not included in the accompanying diagram.

OBSERVATIONS ON AMEBA FEEDING ON INFUSORIA, AND THEIR BEARING ON THE SURFACE-TENSION THEORY

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A considerable number of investigators hold that movement in Ameba is produced exclusively by changes in surface tension. Among the most prominent of these we may mention Bütschli (1892), Ryder (1894), Jensen (1905), and Verworn (1909). Many others, *e.g.*, Rhumbler (1910) and McClendon (1911), hold that while surface tension may not be the only factor involved in the process in question, it certainly is one of the most important. The results of our observations do not support these contentions.

Certain amebae at times feed almost exclusively on rotifers, at others they feed largely on paramecia. They capture the rotifers by flowing around the foot at the point of attachment to the substratum. After they have surrounded the foot they begin to flow out over the body. The rotifer responds by contracting and forcing the ameba back, after which it extends again and the ameba again begins to flow out over it, etc. In the meantime the foot begins to digest and gradually the rotifer weakens. Thus they continue sometimes for days before the rotifer is swallowed. The whole process is of such a nature that after observing it we were fully convinced that the force exerted by the amebae was far greater than could possibly be produced by changes in surface tension. The following observations prove this conviction to be correct with reference to the process of feeding on Paramecium.

When amebae are feeding on paramecia they assume a sort of mushroom shape with a serrate edge consisting of numerous short pseudopods. The paramecia tend to come to rest between and under these pseudopods by which they are usually surrounded, but sometimes the ends of the pseudopods approach each other before they are fully extended and cut the paramecium in two. This process requires approximately only ten seconds.

To cut a paramecium in two with a fine glass fiber it requires a pressure of approximately 9 mgm. Consequently, if the pseudopods have the same cutting quality as the glass fiber and if their movement is due to a change of surface tension, it would require, to perform the work involved, a reduction in surface tension of at least 1118 dynes per centimeter at the tips of the pseudopods. But if the pseudopods fuse along the edges near the ends so as to form a ring around the para-

mecium, and the cutting is due to constriction in this ring, and the constriction to a change in surface tension, the work invo ved would require a minimum reduction along the inner surface of the ring of at least 383 dynes per centimeter.

The bulk of evidence at hand seems to indicate that the paramecia are divided by the approach of two pseudopods and not by the constriction of a ring. To account for the process on the basis of the surface tension theory, therefore, the surface tension of the amebae would, in all probability, have to be considerably higher than 1118 dynes per centimeter. The surface tension of protoplasm is, however, only approximately 50 dynes per centimeter. It is, therefore, probably at best an insignificant factor in the process of feeding in Ameba.

More detailed descriptions of these observations and calculations will be published elsewhere.

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THE ELECTROMOTIVE FORCE PRODUCED BY THE ACCELERATION OF METALS

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Modern theories of electricity have led to the belief that the passage of an electric current through a metal really consists in the progressive motion of 'free' electrons contained in the body of the metal itself. If this be true we may now expect a number of effects arising from the *mass* of these electrons which were not predictable on the basis of older theories which thought of electricity as a sort of intangible massless fluid. As examples of such effects, we should expect the rear end of an *accelerated* rod of metal to become negatively charged owing to the lagging behind of the relatively mobile electrons which the metal contains,

and should expect the periphery of a *rotating* disk to become negatively charged owing to the action of centrifugal force on the electrons in the disk. Such effects would presumably be very small, however, owing to the exceedingly small mass probably associated with the electron.

In the case of electrolytes, where the mass associated with the carriers of electricity is much larger than in metals, such effects have already been detected by Colley[1] and Des Coudres[2] using respectively the accelerational and the centrifugal method, and have been subjected to a more elaborate quantitative investigation by Tolman[3] and by Tolman and Osgerby.[4]

In the case of metallic conductors, Maxwell,[5] himself, was the first, not only to discuss the nature of the phenomena which would arise if mass of the ordinary kind should be associated with electricity in metals, but also to try experiments to detect the possible effects. He had, however, no means of predicting the presumable size of such effects if they did exist, since this was before the development of the electron theory, and he merely states the negative results of his experiments without any information as to the dimensions or efficiency of his apparatus. Lodge[6] also reports a negative result for such experiments.

The first attempts to detect such effects in metals of which we have any quantitative information were made by Nichols[7] in 1906. He employed the centrifugal method, using a rotating aluminum disk and making a rubbing contact at the periphery and center with wires, which led to the electrical measuring apparatus. Such rubbing contacts, in particular the one at the rapidly moving periphery, necessarily introduce larger and variable electromotive forces; nevertheless from a series of experiments Nichols was able to conclude that the mass of the carrier in metals is less than that of the hydrogen atom.

Since in the case of metals the centrifugal method of attack almost necessarily involves some disastrous form of rubbing contact, we have been at work during the last four years developing the accelerational method of attack. In 1913[8] we were able to report that the effect in metals, if any, was so small that the mass of the carrier in metals was less than one two-hundredth part of that of the hydrogen atom. With the help of a much more sensitive galvanometer and eliminating one by one a number of accidental effects which appear when greater sensitiveness is reached, we have now apparently obtained a real effect due to the mass of the carrier in metals.

The apparatus consisted essentially of a coil of insulated copper wire, wound on the periphery of a wheel which could be rotated at a speed of about 5000 r.p.m. and brought suddenly to rest. The two ends of the

coil were lead to the center of the wheel and there made connection, through wires which were allowed to twist up, with a ballistic galvanometer which measured the pulse of electric current which was produced by the tendency of the electrons to continue in motion after the wheel was stopped. For the purpose of eliminating accidental effects it was found among other things necessary to construct the apparatus of non-magnetic materials, to rotate the coil of wire in a space in which both the horizontal and vertical components of the earth's steady magnetic field has been neutralized, and to compensate for the variable part of the earth's magnetic field by connecting in series with the rotating coil a compensating coil having the same flux area but wound in the opposite direction. For details of the apparatus our complete article which has been submitted to the *Physical Review* for publication must be consulted.

The galvanometer throws were found to be in the direction predicted on the basis of a negative charge for the *mobile* carrier in metals and to agree within the limits of error with the equation

$$Q = M\frac{vl}{RF},$$

of which the derivation will be given in our complete article. Q is the pulse of electricity sent through the galvanometer on stopping, v is the velocity of the periphery of the wheel at the instant of stopping, l the length of the coil, R the total resistance in the circuit, F the value of the faraday, and M is a constant whose value ought to be fairly close to that of the mass associated with one equivalent (*i.e.*, with $F = 96{,}540$ coulombs) of electrons. For copper wire the average value of M from 131 runs was 1/1910 which may be compared with the accepted value for the mass of one equivalent of electrons in free space, namely 1/1845 (*i.e.*, the ratio of the mass of the electron to that of the hydrogen atom).

Our purpose in carrying out these measurements has not been merely to demonstrate an effect which has long been an object of search; we have also had in mind the possibility of obtaining from our experiments information as to the nature of the conducting process in metals and indeed perhaps further information as to the nature of the electron itself.The equation given above which we have tested in this work was derived on the assumption that the conducting process in metals is in the nature of a drift of 'free' electrons when acted on by an electric field, and the fact that the equation seems to fit the experimental facts is to some extent a verification of these assumptions. Such considerations are of particular interest at the present time in view of J. J. Thomson's pro-

posal[9] of a quite different theory of metallic conduction. According to his theory, a metal contains atoms which are in the nature of electrical doublets which will orient themselves parallel to any applied electrical field. These atoms are assumed to have the power of ejecting electrons in the same direction as the axis of the doublet and hence the conducting process on the basis of this theory consists in a tendency for orientation of the doublets under the action of the applied electromotive force and a consequent ejection of electrons from one atom to another in the direction in which the current is known to flow. It seems very doubtful to us whether such a theory can be satisfactorily brought into agreement with our experimental results, since it would seem at first sight to be merely an accidental coincidence if the *mechanical* forces which we apply should produce an orientation in the right direction and of the right amount to give the pulse of electricity whose magnitude we have calculated on the basis of the other theory and actually found experimentally.

[1] Colley, *Ann. Physik., Leipzig*, **17**, 55 (1882).
[2] Des Coudres, *Ibid.*, **49**, 284 (1893); *Ibid.*, **57**, 232 (1896).
[3] Tolman, *Proc. Amer. Acad. Arts Sci.*, **46**, 109 (1910); *J. Amer. Chem. Soc.*, **33**, 121 (1911).
[4] Tolman, Osgerby and Stewart, *J. Amer. Chem. Soc.*, **36**, 466 (1914).
[5] Maxwell, *Treatise on Electricity and Magnetism*, 3rd edition (1892), Vol. 2, pp. 211 et seq.
[6] Lodge, *Modern Views of Electricity*, 3rd edition (1907), p. 39.
[7] Nichols, *Physik. Zs.*, **7**, 640 (1906).
[8] Tolman, Osgerby and Stewart, loc. cit.
[9] Thomson, *Phil. Mag.*, **30**, 192 (1915); Richardson, *Ibid.*, **30**, 295 (1915).

Errata: In Mr. T. W. Vaughan's article, pages 98 and 99, the familiar percentage sign (%) was printed in place of the per-thousandths sign (‰), following figures for the salinity; the salinities as printed are therefore ten times too large.